Wolfgang Renner
Gimbacher Weg 34
D-65779 Kelkheim
www.institut-wolfgang-renner.de
institut-wolfgang-renner@t-online.de
Telefon: 06195 4161

Kelkheim den 11. März 2015

Werner-Wheeler Masses for a Clusterdecay-Model

Retrospective view on fission-science history:

1896 Becquerel discovered radioactivity.

1908 Rutherford showed that α-particles are helium-nuclei.

1912 empirical Geiger-Nuttall Rule: $\ln(\lambda) = Const + b \cdot E$

1928 With the new Quantum-mechanics Gamov explained theoretically α-decay and Geiger-Nuttall Rule with "tunneling".

1939 Hahn, Meitner and Strassmann discovered Uranium-fission.

1939 N. Bohr theoretically investigated fission with the Bethe-Weizsäcker Liquid-drop model and Legendre-expansion geometry:

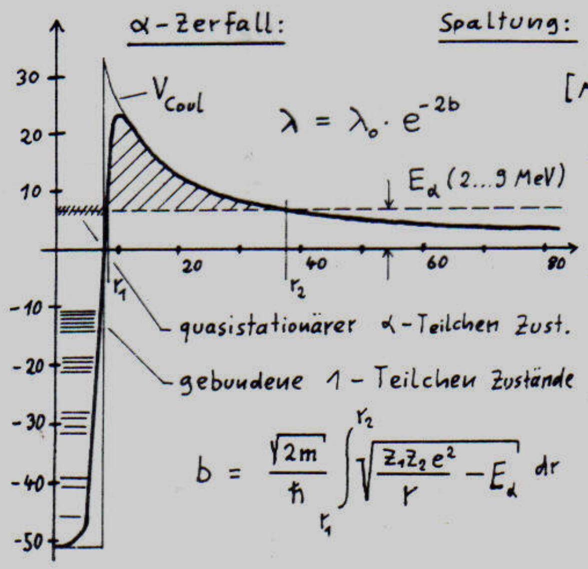
$$R(\theta) = R_0 \left[1 + \alpha_0 + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + \dots \right]$$

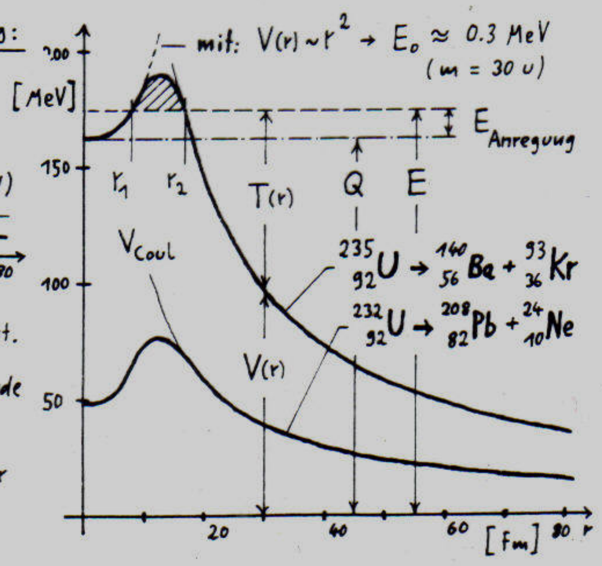
This Led to the scission-parameter $x = \frac{1}{2} E_C / E_S$. If $x < 1$ then the sphere-form is a local energy-minimum (stabel). Besides Bohr sketched the PES (Potential-Energy-Surface)

1947 Frankel calculated Bohr's PES on the Eniac.

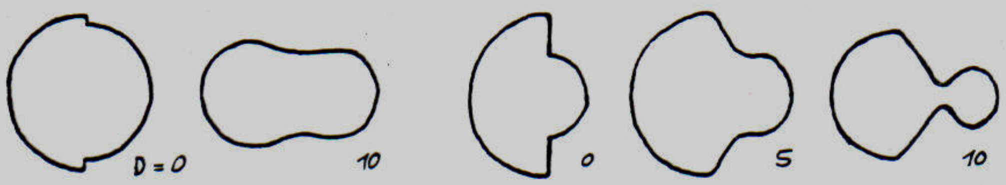
1965 J. Lawrence used the geometry: $\rho^2(z) = A \cdot z^4 + B \cdot z^2 + C$

1969 J. Nix introduced a geometry of two ellipsoids smoothly connected with a hyperboloid Neck. It has 6 gen. coord.





≈1970 For various fission models single-particle shell corrections are calculated. For example J.A. Maruhn investigated the mass-asymmetry with an advanced Holzer-oszillator. The geometry was two ellipsoids connected with a polynomial function



$^{235}_{92}U \rightarrow ^{93}_{36}Kr + ^{140}_{56}Ba + 2n$ $\qquad$ $^{232}_{92}U \longrightarrow ^{24}_{10}Ne + ^{208}_{82}Pb$

1980 From this kind of calculations A. Sandulescu, D.N. Poenaru and W. Greiner predicted very asymmetric fission.

1984 Rose and others discovered this "Cluster-decays" experimentally. For example: $^{223}Ra \rightarrow ^{14}C + ^{209}Pb$ or $^{232}U \rightarrow ^{24}Ne + ^{208}Pb$.

≈1987 Because the old geometries are not very suitable for describing very asymmetric fission, Hou-ij Wang invented the new (TBM) Three-Ball-Model. It gives up to α-Decay good shapes.

1988 K. Depta calculated the PES of the new geometry with a simple and an advanced Yukawa-plus-exponential macro. formal. D. Schnabel calculated the microscopic shell-correction

1989 Now the calculation of the hydrodynamic Werner-Wheeler masses for the TBM-model are presented.

For example the masses are needed for Livetime calculations:

$$N(t) = N_0 \cdot e^{-\lambda t} \qquad b = \int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar^2}\left(V(s) - E\right) \cdot M(s)}\, ds \qquad M(s) = \sum_{ij} M_{ij} \frac{\partial q_i}{\partial s} \frac{\partial q_j}{\partial s}$$

$$\lambda = \lambda_0 \cdot e^{-2b}$$

Where the penetration-integral b is obtained from the WKB-approximation. The q_i are the generalized coordinates of the model.

The Werner-Wheeler Formalism

Collective nuclear models are defined in generalized coordinates q_i, $i=1...n$. Therewith the Lagrangian is given by: $L(q,\dot{q}) = \sum_{i,j=1}^{n} M_{ij}\,\dot{q}_i\dot{q}_j + V(q)$

The massmatrix M_{ij} is unknown and can be determined from the kinetic energy of the flowing liquid:

$$T = \frac{M}{2}\int_{\text{shape}} \vec{V}^2(\vec{r})\,d^3r \qquad \vec{V}(\vec{r}) = V_x(x,y,z)\cdot\vec{e}_x + V_y\cdot\vec{e}_y + V_z\cdot\vec{e}_z = V_z(z,\varrho,\varphi)\cdot\vec{e}_z + V_\varrho\cdot\vec{e}_\varrho + V_\varphi\cdot\vec{e}_\varphi$$

The velocityfield consists of three functions with three arguments. To determine it we need 9 functional relations:

1. Relation Incompressibility: $\vec{\nabla}\cdot\vec{V} + \frac{\partial\varrho}{\partial t} = \vec{\nabla}\cdot\vec{V} = 0$

2.-6. Relation Cylindersymmetry: $\vec{V}(z,\varrho,\varphi) \rightarrow V_z(z,\varrho)\cdot\vec{e}_z + V_\varrho(z,\varrho)\cdot\vec{e}_\varrho$

Now some physical considerations must be taken into account. The demand of rotationless flow lead to minimal kinetic energy and respectively masses.

7.-8. Relation "Irrotational-Flow Model": $\vec{\nabla}\times\vec{V} = 0 \rightarrow \vec{V} = \vec{\nabla}\phi_v(z,\varrho)$

Alternatively one can simplify the field in an arbitrary but appropriate manner which makes proper and easy calculation possible:

7.-8. Relation "Werner-Wheeler Model": $V_z(z,\varrho) \rightarrow V_z(z)$ and $V_\varrho(z,\varrho) \rightarrow \varrho\cdot\tilde{V}_\varrho(z)$

To eliminate the last degree of freedom and determine the field complet-ly one has to work in the geometry which is given by the Radius $P(z)$

9. Relation Geometry: $\vec{V}(z,\varrho) = \vec{V}(z,\varrho,P(z))$

To obtain the masses from the expression of the kinetic energy, the velocityfield must be given as linear functions from the generalized velocities:

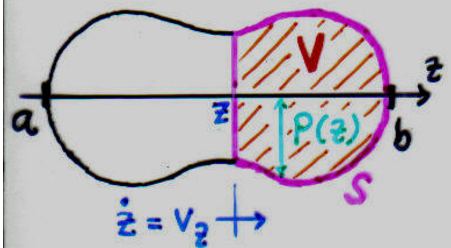
$$V_z(z) = \sum_{i=1}^{n} A_i(z,q)\cdot\dot{q}_i \quad \text{and} \quad V_\varrho(z,\varrho) = \varrho\sum_{i=1}^{n} C_i(z,q)\cdot\dot{q}_i \quad \left(= \frac{\varrho}{P(z)}\sum_{i=1}^{n} B_i(z,q)\cdot\dot{q}_i\right)$$

By expressing the Nabla-operator in cylindercoordinates the continuity equation leads to a relation between $C_i(z,q)$ and $A_i(z,q)$:

$$\vec{\nabla}\cdot\vec{v} = \frac{\partial}{\partial z} v_z(z) + \frac{1}{\varrho}\frac{\partial}{\partial \varrho}(\varrho\cdot v_\varrho) = \sum_{i=1}^{n}\left(\frac{\partial}{\partial z}A_i + \frac{1}{\varrho}\frac{\partial}{\partial \varrho}(\varrho^2\cdot C_i)\right)\cdot \dot{q}_i$$

$$\vec{\nabla}\cdot\vec{v} = \sum_{i=1}^{n}\left(\frac{\partial}{\partial z}A_i + 2\cdot C_i\right)\cdot\dot{q}_i = 0 \quad\rightarrow\quad C_i = -\frac{1}{2}\frac{\partial}{\partial z}A_i$$

For the calculation of the A_i we imagina a surface S which encloses an constant volume V and is moving with the liquid. When it is placed like in the picture shown it results in :



$$V(z,q) = \pi\int_z^b P^2(z',q)\,dz' \qquad \frac{dV}{dt} = \frac{\partial V}{\partial z}\cdot\dot{z} + \sum_{i=1}^{n}\frac{\partial V}{\partial q_i}\cdot\dot{q}_i$$

with $\frac{\partial}{\partial z}\left(\pi\int_z^b P^2(z')\,dz'\right) = -\pi P^2(z)$ one obtains:

$$\frac{dV}{dt} = \pi\left\{-P^2\cdot\dot{z} + \sum_{i=1}^{n}\frac{\partial}{\partial q_i}\left(\int_z^b P^2(z')\,dz'\right)\cdot\dot{q}_i\right\} = 0 \quad\rightarrow\quad v_z = \frac{1}{P^2}\sum_{i=1}^{n}\left(\int_z^b P^2(z')\,dz'\right)\cdot\dot{q}_i$$

By comparing this result with the given definition of the A_i one can immediately see (The second form goes similar and gives the same result):

$$A_i = \frac{1}{P^2(z)}\frac{\partial}{\partial q_i}\int_z^b P^2(z',q)\,dz' \qquad \text{and} \qquad A_i = -\frac{1}{P^2(z)}\frac{\partial}{\partial q_i}\int_a^z P^2(z',q)\,dz'$$

Now with this $\vec{v}$-definition we calculate the kinetic energy :

$$T = \frac{\sigma}{2}\int_{V_0}\vec{v}^{\,2}\,dV \overset{!}{=} \frac{1}{2}\sum_{i,j=1}^{n} M_{ij}\,\dot{q}_i\dot{q}_j \qquad \text{Mit: } \vec{v}^{\,2} = v_z^2(z) + v_\varrho^2(z,\varrho) \qquad \sigma = \frac{M}{V_0}$$

$$T = \frac{\sigma}{2}\int_0^{2\pi}\int_a^b\int_0^{P(z)}\left\{\left(\sum_i A_i\dot{q}_i\right)\left(\sum_j A_j\dot{q}_j\right) + \left(\varrho\sum_i C_i\cdot\dot{q}_i\right)\left(\varrho\sum_j C_j\dot{q}_j\right)\right\}\varrho\,d\varrho\,dz\,d\varphi$$

$$T = \frac{1}{2}\sigma\pi\sum_{i,j=1}^{n}\int_a^b\left\{A_iA_j P^2 + C_iC_j\frac{P^4}{2}\right\}dz\cdot\dot{q}_i\dot{q}_j$$

In this expression one immediately recognize the elements of M_{ij} :

$$M_{ij} = \sigma\pi\int_a^b P^2\left\{A_iA_j + \frac{P^2}{2}C_iC_j\right\}dz$$

The Treatment of the center of mass movement

To make accurate physics one has to take the center-off-mass conservation into consideration. Center of mass is given by:

$$z_s(q) = \frac{\pi}{V_0}\int_a^b P^2(z,q)\, z\, dz \qquad \dot{z}_s(q) = \sum_i \frac{\partial z_s}{\partial q_i}\cdot \dot{q}_i = \sum_i Q_i \cdot \dot{q}_i$$

To treat this z_s-influence it would be possible to transform the geometry to the z_s-system: $P(z) \rightarrow P(z-z_s) = \tilde{P}(z)$

But this makes the calculation off the masses rather impossible.

The easiest alternative way is, to calculate the mass-matrix MS_{ij} of the center of mass movement separated and then subtract it from the not corrected WW-matrix M_{ij} which yields the correct MA_{ij}-matrix:

$$T_s = \frac{1}{2} M \dot{z}_s^2 = \frac{1}{2} M \Big(\sum_i Q_i \dot{q}_i\Big)\Big(\sum_j Q_j \dot{q}_j\Big) = \frac{1}{2}\sum_{ij} MS_{ij}\cdot \dot{q}_i \dot{q}_j$$

$$MS_{ij} = M \cdot Q_i Q_j \qquad\qquad MA_{ij} = M_{ij} - MS_{ij}$$

An other possible way is, to subtract the center of mass velocity from the z-velocity and then applying the WW-formalism:

$$V_z \rightarrow V_z - \dot{z}_s = \sum_i A_i \dot{q}_i - \sum_i Q_i \dot{q}_i = \sum_i (A_i - Q_i)\cdot \dot{q}_i$$

$$T = \sigma\pi \int_a^b \int_0^{P(z)} \left((V_z - \dot{z}_s)^2 + V_\varrho^2\right)\varrho\, d\varrho\, dz = \sum_{ij} \frac{\sigma\pi}{2}\int_a^b P^2 \left\{(A_i - Q_i)(A_j - Q_j) + \frac{P^2}{2} C_i C_j\right\} dz \cdot \dot{q}_i \dot{q}_j$$

$$\Rightarrow MA_{ij} = \sigma\pi \int_a^b P^2 \left\{A_i A_j + \frac{P^2}{2} C_i C_j\right\} dz - \sigma\pi\left[Q_j M Q_i + Q_i M Q_j\right] + \sigma Q_i Q_j\, \pi \int_a^b P^2 dz$$

$$MA_{ij} = M_{ij} - MS_{ij} \qquad \text{with:} \quad \pi\int_a^b P^2 A_i\, dz = V_0 \cdot Q_i \quad \text{and} \quad \pi \int_a^b P^2 dz = V_0$$

One obtains the same result. With this second approach one can calculate MS_{ij} as $MS_{ij} = M_{ij} - MA_{ij}$. Comparing this MS_{ij} with the direct calculated one, is an effective method for finding bugs in the program.

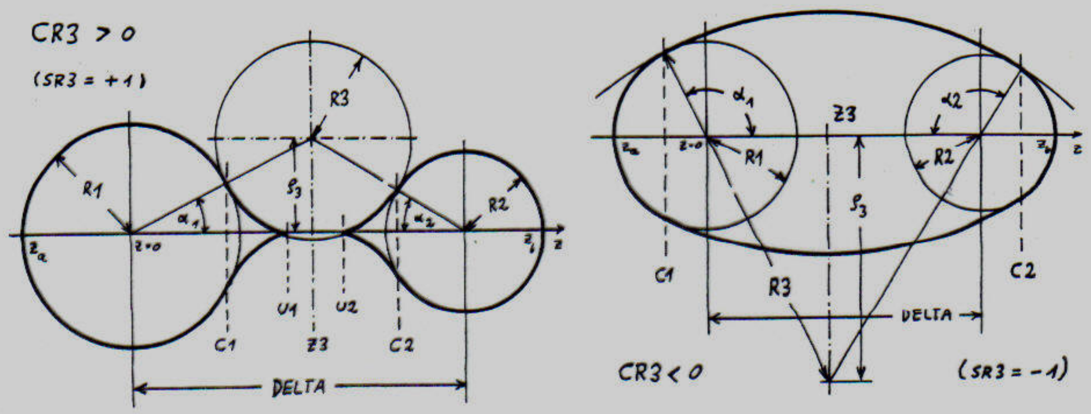
The Geometry of the Three-Ball Model

This new nuclear-shape parametrization has 3 generalized coordinates:

1.) $ETA = \dfrac{A1 - A2}{A1 + A2}$ mass-asymmetry

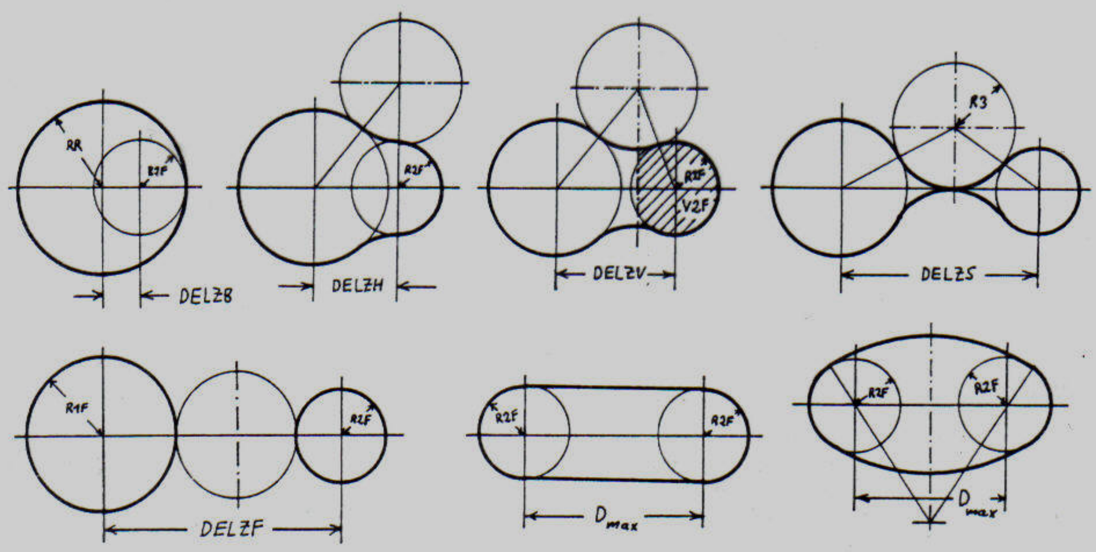
2.) DELTA centerdistance of the fragmentballs

3.) $CR3 = \dfrac{SR3}{R3}$ curvature of the neck



DELTA is the main fragmentation coordinate. There are some special points in DELTA with a special meaning. The scission-point is called DELZS and the final point DELZF. After DELZS the geometry is fixed. But before, there is some freedom in choice of geometry in spite of volume conservation.

One possibility is this α-decay like form which I call TBM:



(a)

-6-

There are now three special points more : DELZB, DELZH, DELZV

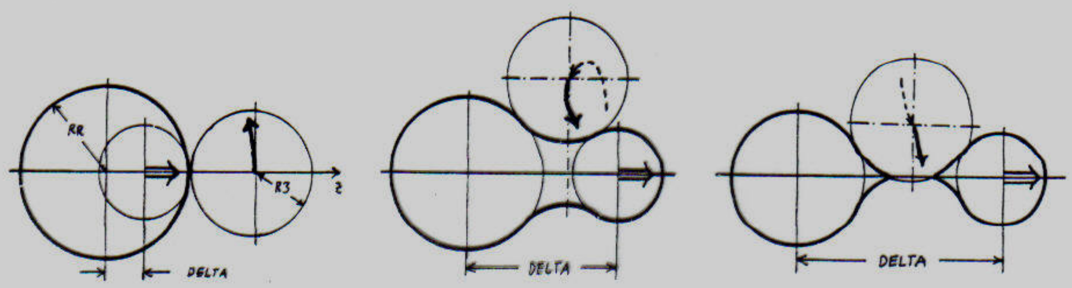
advantages: $R2 \approx R2F = \left(\frac{3}{4\pi} V2F\right)^{1/3}$, $V2F = \frac{V_0}{2}(1-ETA)$ α-decay Like

Potential Energy Surface PES Looks O.K.

disadvantages: Shapes and all results asymmetric in ETA

kinks in the WW-masses due to definition-change at DELZV

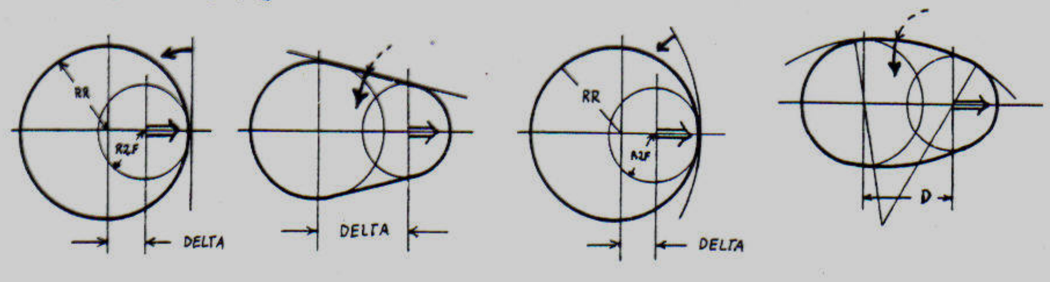
- To illustrate the movement of the TBM model
- Look to this picture :



positiv neck-curvature CR3

cone - case

negativ neck-curvature CR3



Attempts for a better geometry

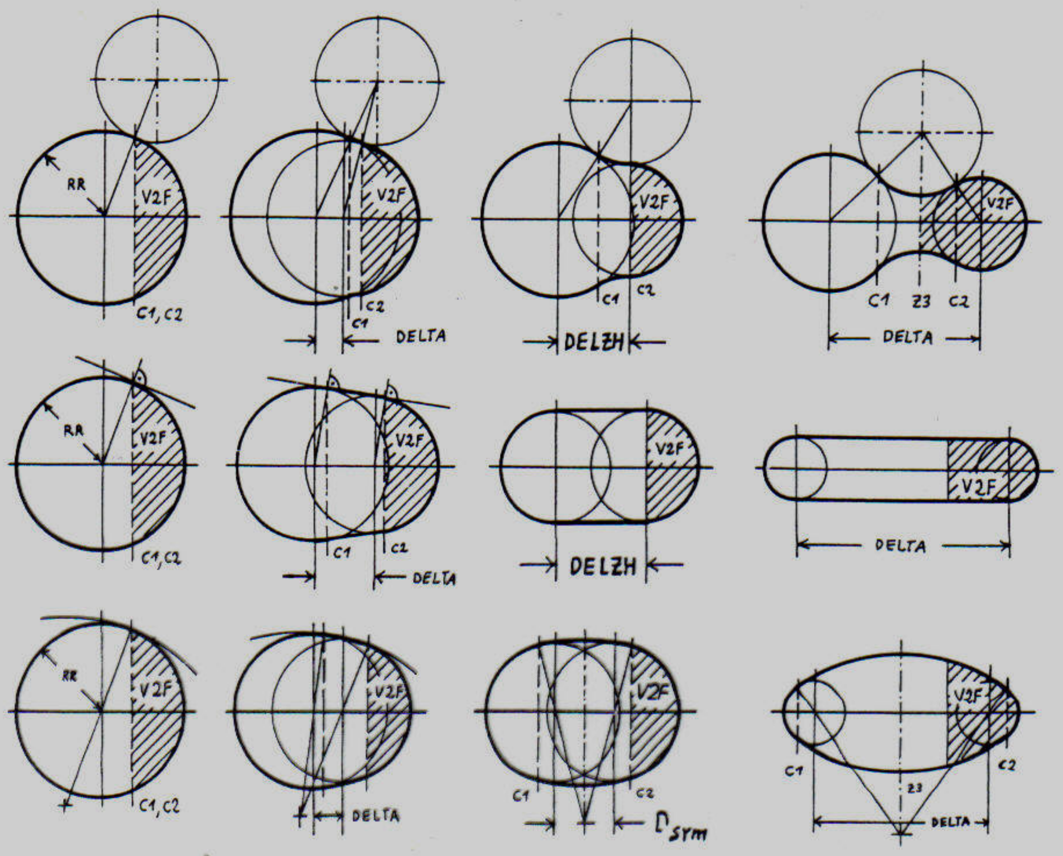
To get an optimal behavior between the beginning and DELZS the symmetry requirement suggests to divide the nucleus from the beginning with an imaginary plane into the final volume ratio. This Leads indeed to ETA-symmetry but there must be also a change in definition which again produces the unwanted kinks.

The new definition :

1.) The dividing plane is kept at C2 until DELZH.

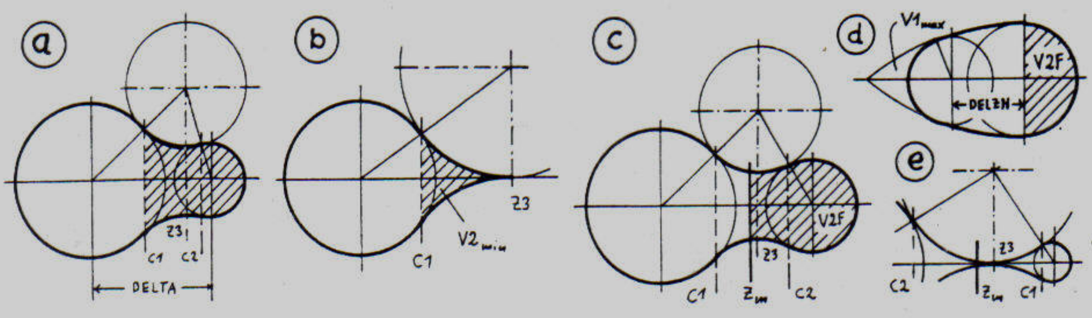
2.) Then it is given to R3 until the end.

The new ETA-symmetric geometry has also a definition change at DELZH when R3 crosses C2 during the crowing of DELTA :



To avoid the crossing of R3 with C2 one could think it is better to use C1 as place of the dividing plane. But this leads to useless configurations (Picture ⓐ and ⓑ).

In general near to the scission point R3 must become to the dividing plane. It lays there between C1 and C2. This suggests to put the plane from the beginning between C1 and C2. Put it in the middle ⓒ will make trouble at the scissionpoint ⓔ. To avoid this a more sophisticated function would be necessary. But this makes WW-calculation impossible.

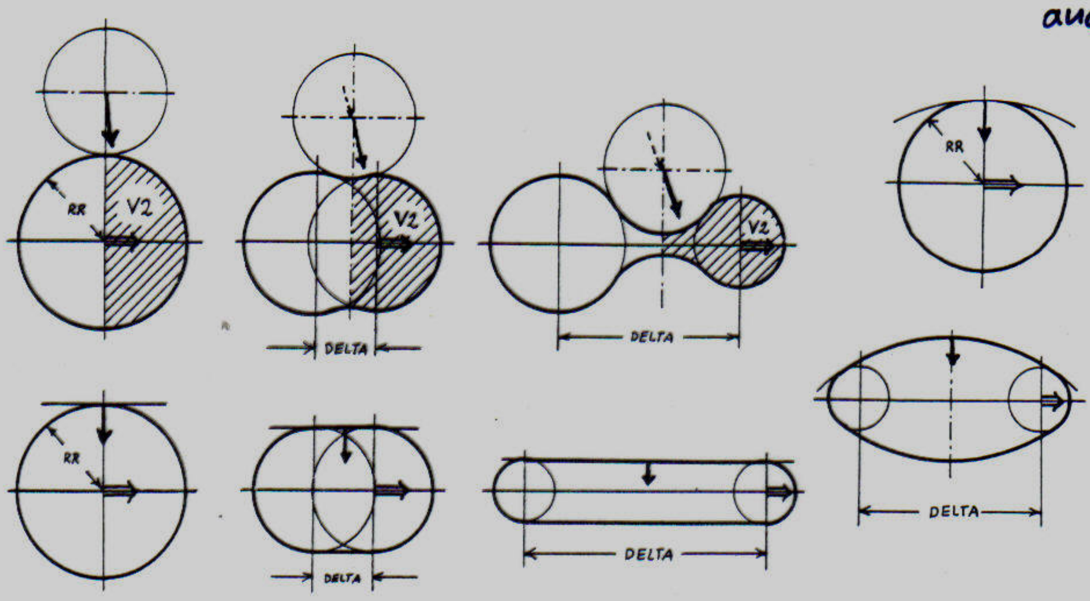


©

DKM or success with the fourth effort

In the first model "TBM" the fragmentradius R2 was kept near R2F to obtain an α-decay like configuration. This led to the ETA-asymmetry. To avoid this asymmetry in the new models the volume was divided from the beginning into the final ratio. The particular geometry was then given by the location of the dividing plane. The problems with this were shown.

To avoid the kinks it seems unavoidable to define the geometry smooth from the beginning until scission. At scissionpoint Z3 must be dividing plane. Now Z3 is it from the beginning and it must always remain between C1 and C2:



Now this smooth definition makes it necessary to make the volume V2 to a smooth function. It must be half of V_0 at the beginning and V2F at scission. Furthermore Z3 always must remain between C1 and C2. Finaly the derivation $\left.\frac{\partial V2}{\partial DELTA}\right|_{DELZS} \overset{!}{=} 0$ at scission has to be zero to get a smooth scission point behavior. The simplest function is a parabel:

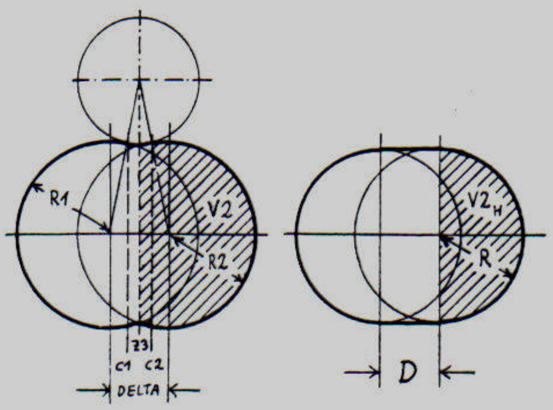
$$V2(DELTA) = \frac{V_0}{2}\left(1 - \left\{1 - \frac{(DELTA - DELZS)^2}{DELZS^2}\right\} \cdot ETA\right)$$

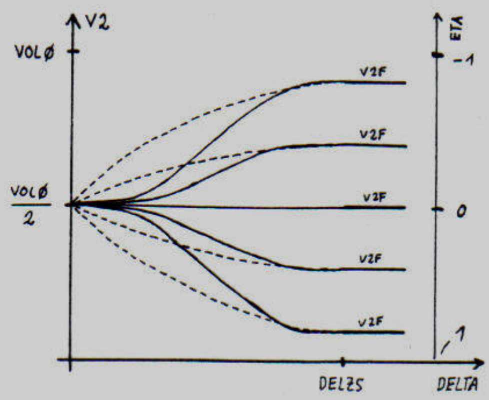
This function is smooth and it's derivation vanishes at scissionpoint.

(d) But will Z3 always remain between C1 and C2 ?

-9-

An simple estimate makes test-programming unnecessary. Problems will appear when V2 decreases so fast, that Z3 must go right of C2. Fore a simple estimate the cone-case is most suited:





There is the critical volume $V2_H$ which must faster decrease than V2(D). The DELTA-derivation of V2(DELTA) at the beginning is given by:

$$\frac{dV_2}{dD}\Big|_{D=0} = -\frac{V_0 \cdot ETA}{DELZS} = -\frac{\frac{4\pi}{3}R^3 \cdot ETA}{DELZS} = -\frac{\pi}{2}R^2 \cdot \frac{8}{3}\frac{R \cdot ETA}{DELZS}$$

And the derivation of $V2_H$ is given by:

$$\frac{dV2_H}{dD}\Big|_{D=0} = \frac{dV2_H}{dR}\cdot\frac{dR}{dD} = 2\pi R^2 \cdot \left(-\frac{1}{4}\right) = -\frac{\pi}{2}R^2$$

$\frac{dR}{dD} = -\frac{1}{4}$ implicitly given by: $V_0(R(D),D) = \pi R^2\left(\frac{4}{3}R + D\right) = 0$

This results in the condition $DELZS > \frac{8}{3} R \cdot DELTA$ which isn't always valid.

So a new attempt was made with a cosine-like function:

$$V2(DELTA) = \frac{V_0}{2}\left(1 - \left\{1 - \frac{1}{2}\left(1 + \cos\left(\frac{\pi \cdot DELTA}{DELZS}\right)\right)\right\} \cdot ETA\right)$$

This function fulfils the estimate and also programmed it gives good shapes. This model is called "DKM". The masses are up to 5 times larger.

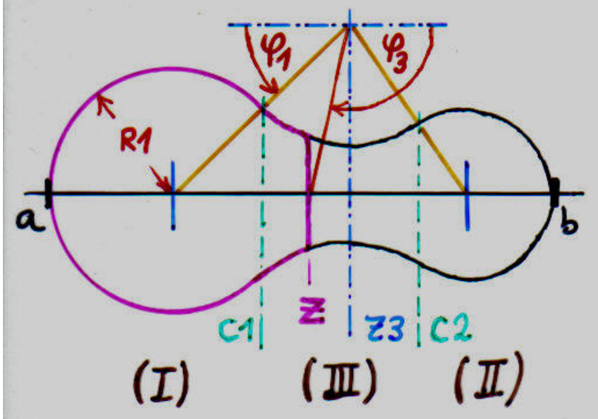
<u>advantages:</u> ETA-symmetric and smooth masses.

<u>disadvantages:</u> Shapes are not α-decay like and PES has a strange valley.

<u>Are there any possibilities more ?</u>

Because it is intended to describe the α-decay similar very asymmetric fission, I tried to find an smooth TBM-like geometry, but with no success.

Werner – Wheeler calculation for the Three – Ball Model



$$X = (x_1, x_2, x_3) = (\mathrm{ETA}, \mathrm{DELTA}, \mathrm{CR3})$$

$$Y = (y_1, y_2, \ldots, y_5) = (R1, \mathrm{KOS1}, \varrho_3, \mathrm{KOS2}, R2)$$

$$\mathrm{KOS1{,}2} = \cos(\varphi_{1,2}) \; ; \; \mathrm{KOS3} = \cos(\varphi_3)$$

$$A_{x_i} = -\frac{1}{P^2_{(z)}} \frac{\partial}{\partial x_i} \int_a^z P^2(z')\,dz' \; ; \; C_{x_i} = -\frac{1}{2} \frac{\partial}{\partial z} A_i$$

Segment (I): $\quad P^2(z) = R1^2 - z^2 \qquad I^{(I)} = \int_{R1}^{z} P^2(z')\,dz' = \frac{2}{3} R1^3 + R1^2 \cdot z - \frac{z^3}{3}$

$$\frac{\partial I}{\partial x_i} = 2 \cdot R1 \cdot (R1 + z) \cdot \frac{\partial R1}{\partial x_i} \;\rightarrow\; A_{x_i} = -\frac{\partial R1}{\partial x_i} \cdot \frac{2\,R1}{R1 - z} \; ; \; C_{x_i} = \frac{\partial R1}{\partial x_i} \cdot \frac{R1}{(R1 - z)^2}$$

Segment (II): $\quad P^2(z) = R2^2 - (D - z)^2 \;\rightarrow\; A_{x_i} = \frac{\partial R2}{\partial x_i} \cdot \frac{2\,R2}{R2 - (D-z)^2} + \delta_{x_i D} \; ; \; C_{x_i} = \frac{\partial R2}{\partial x_i} \cdot \frac{R2}{(R2 - (D-z))^2}$

Segment (III): $\quad P(z) = \varrho_3 - SR3 \cdot \sqrt{R3^2 - (z - z3)^2} \quad ; \; R1, R2 \rightarrow R \; ; \; \mathrm{KOS1}, \mathrm{KOS2} \rightarrow \mathrm{KOS}$

$$I^{(III)} = \frac{R^3}{3}(1 + \mathrm{KOS})^2 \cdot (2 - \mathrm{KOS}) + SR3 \cdot \left(\varrho_3^2 \cdot R3 \cdot (\mathrm{KOS} + \mathrm{KOS3}) + \frac{R3^3}{3}\left[\mathrm{KOS} \cdot (3 - \mathrm{KOS}^2) + \mathrm{KOS3} \cdot (3 - \mathrm{KOS3}^2)\right]\right)$$
$$- \varrho_3 \cdot R3^2 \cdot \{\mathrm{KOS} \cdot \mathrm{SIN} + \mathrm{KOS3} \cdot \mathrm{SIN3} + \arcsin(\mathrm{KOS}) + \arcsin(\mathrm{KOS3})\}$$

$$\frac{\partial I}{\partial x_i} = \frac{\partial I}{\partial R} \cdot \frac{\partial R}{\partial x_i} + \frac{\partial I}{\partial \mathrm{KOS}} \cdot \frac{\partial \mathrm{KOS}}{\partial x_i} + \frac{\partial I}{\partial \varrho_3} \cdot \frac{\partial \varrho_3}{\partial x_i} + \frac{\partial I}{\partial R3} \cdot \frac{\partial R3}{\partial x_i} + \frac{\partial I}{\partial \mathrm{KOS3}} \cdot \frac{\partial \mathrm{KOS3}}{\partial x_i}$$

$$\mathrm{KOS3} = \mathrm{KOS3}(y, x) \;\rightarrow\; \frac{\partial \mathrm{KOS3}}{\partial x_i} = \sum_j \frac{\partial \mathrm{KOS3}}{\partial y_j} \cdot \frac{\partial y_j}{\partial x_i} + \delta_i \;\rightarrow\; \frac{\partial I}{\partial x_i} = \sum_j F_j(y, x) \cdot \frac{\partial y_j}{\partial x_i} + \tilde{\delta}_i$$

$$A_{x_i} = -\frac{1}{P^2} \frac{\partial}{\partial x_i} I = \sum_j \tilde{F}_j(y, x) \cdot \frac{\partial y_j}{\partial x_i} + \tilde{\delta}_i \; ; \; C_{x_i} = \sum_j G_j(y, x) \cdot \frac{\partial y_j}{\partial x_i}$$

Center of mass Movement

$$z_S = \frac{\pi}{V_0} \int_a^b P^2(z) \cdot z \, dz \quad ; \quad \dot{z}_S = \sum_i \frac{\partial z_S}{\partial x_i} \cdot \dot{x}_i = \sum_i Q_i \cdot \dot{x}_i$$

$$Q_i = \frac{\partial z_S}{\partial x_i} = \sum_j \frac{\partial z_S}{\partial y_j} \cdot \frac{\partial y_j}{\partial x_i} = \sum_j H_j(y, x) \cdot \frac{\partial y_j}{\partial x_i}$$

-11-

The calculation of the partial derivations $\frac{\partial y_j}{\partial x_i}$ –

The geometry was numericaly calculated and is given in the quantities y_j. Of course the y_j are functions of the generalized coordinates : $y_j = y_j(x)$. The 5 functions $y_j(x)$ are not explicitly known. Nevertheless we need the derivations $\frac{\partial y_j}{\partial x_i}$ which can be calculated with the law about implicit functions:

$$\left(\frac{\partial y}{\partial x}\right) = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_3} \\ \vdots & & \\ \frac{\partial y_5}{\partial x_1} & \cdots & \frac{\partial y_5}{\partial x_3} \end{pmatrix} = -\left(\frac{\partial F}{\partial y}\right)^{-1} \cdot \left(\frac{\partial F}{\partial x}\right) = -\begin{pmatrix} \frac{\partial F_1}{\partial y_1} & \cdots & \frac{\partial F_1}{\partial y_5} \\ \vdots & & \vdots \\ \frac{\partial F_5}{\partial y_1} & \cdots & \frac{\partial F_5}{\partial y_5} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial x_3} \\ \vdots & & \\ \frac{\partial F_5}{\partial x_1} & \cdots & \frac{\partial F_5}{\partial x_3} \end{pmatrix}$$

Now we need the 5 implicit functions $F_1(y,x) = 0, \cdots, F_5(y,x) = 0$. The volume conditions give the first to functions. They are different for the two geometry-models TBM and DKM. First we consider TBM:

$$F_1 = \mathrm{Volume}(y,x) - V_0 = 0 \; ; \; F_2 = \frac{4\pi}{3} R2^3 - \frac{V_0}{2}(1 - ETA) = 0 \qquad \begin{matrix} DELTA \leq DELZV \\ \text{or } CR3 < 0 \end{matrix}$$

$$F_1 = V1(y,x) - \frac{V_0}{2}(1+ETA) = 0 \; ; \; F_2 = V2(y,x) - \frac{V_0}{2}(1-ETA) = 0 \qquad DELTA > DELZV$$

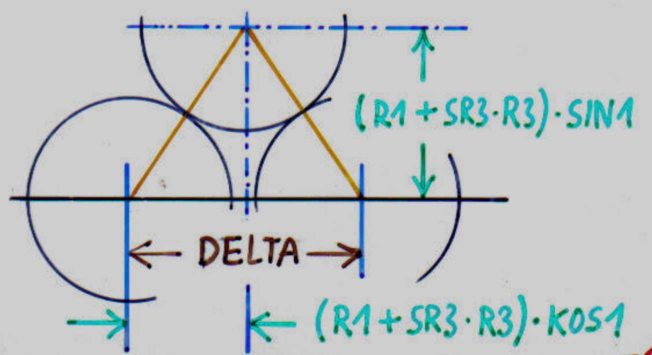
Now DKM:

$$F_1 = V1(y,x) - \frac{V_0}{2} = 0 \; ; \; F_2 = V2(y,x) - \frac{V_0}{2} = 0 \qquad CR3 < 0$$

$$\left.\begin{matrix} F_1 = V1(y,x) - \frac{V_0}{2}\left(1 + \left\{1 - \frac{1}{2}\left(1 + \cos\left(\frac{\pi \cdot DELTA}{DELZS}\right)\right)\right\} \cdot ETA\right) = 0 \\ F_2 = V2(y,x) - \frac{V_0}{2}\left(1 - \left\{1 - \frac{1}{2}\left(1 + \cos\left(\frac{\pi \cdot DELTA}{DELZS}\right)\right)\right\} \cdot ETA\right) = 0 \end{matrix}\right\} \; DELTA \leq DELZS$$

$$F_1 = V1(y,x) - \frac{V_0}{2}(1+ETA) = 0 \; ; \; F_2 = V2(y,x) - \frac{V_0}{2}(1-ETA) = 0 \qquad DELTA > DELZS$$

The remaining 3 functions are given from general geometry conditions:



Smooth Neck – connection gives:

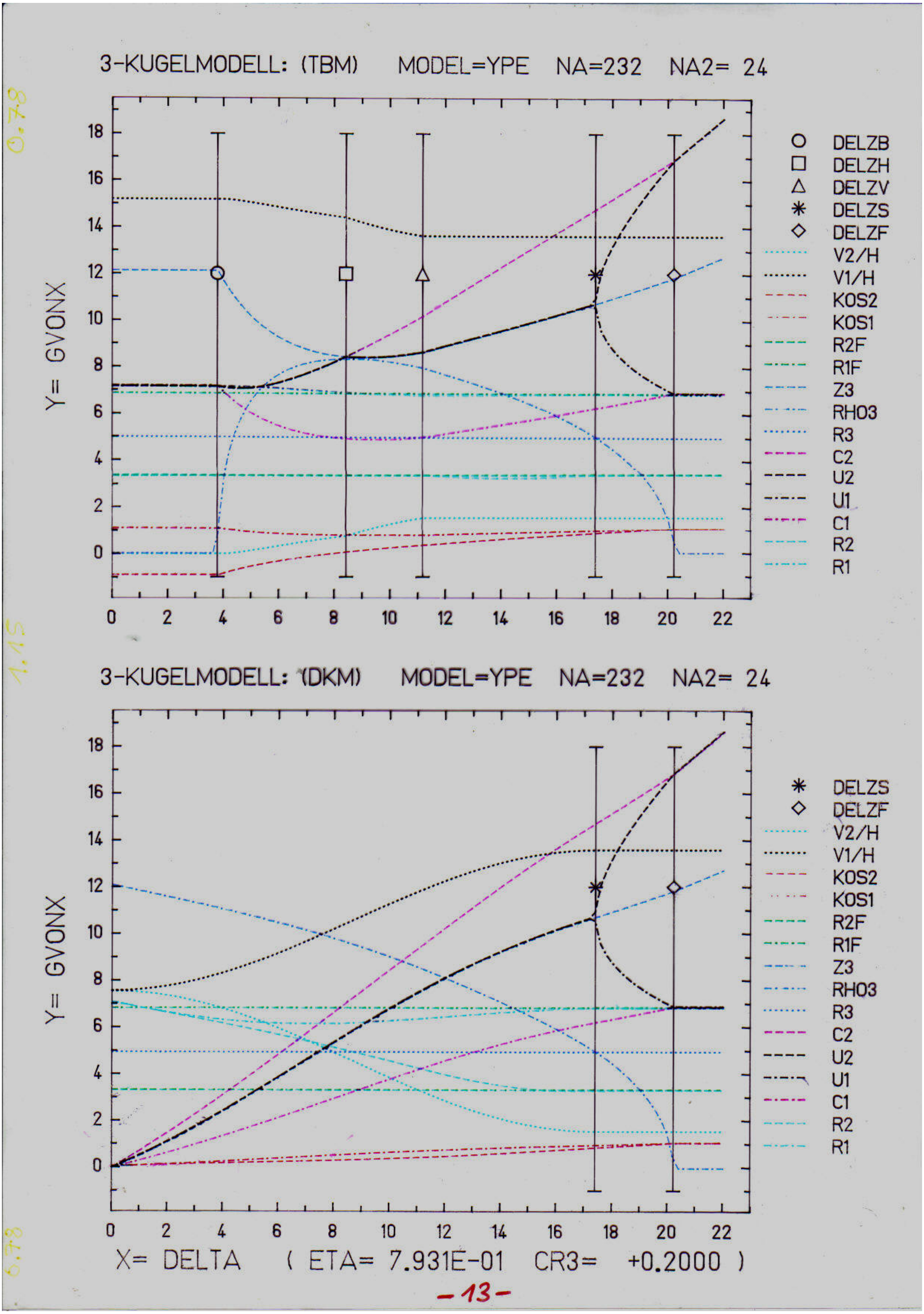
$$F_3 = (R1 + SR3 \cdot R3) \cdot SIN1 - \rho_3 = 0$$

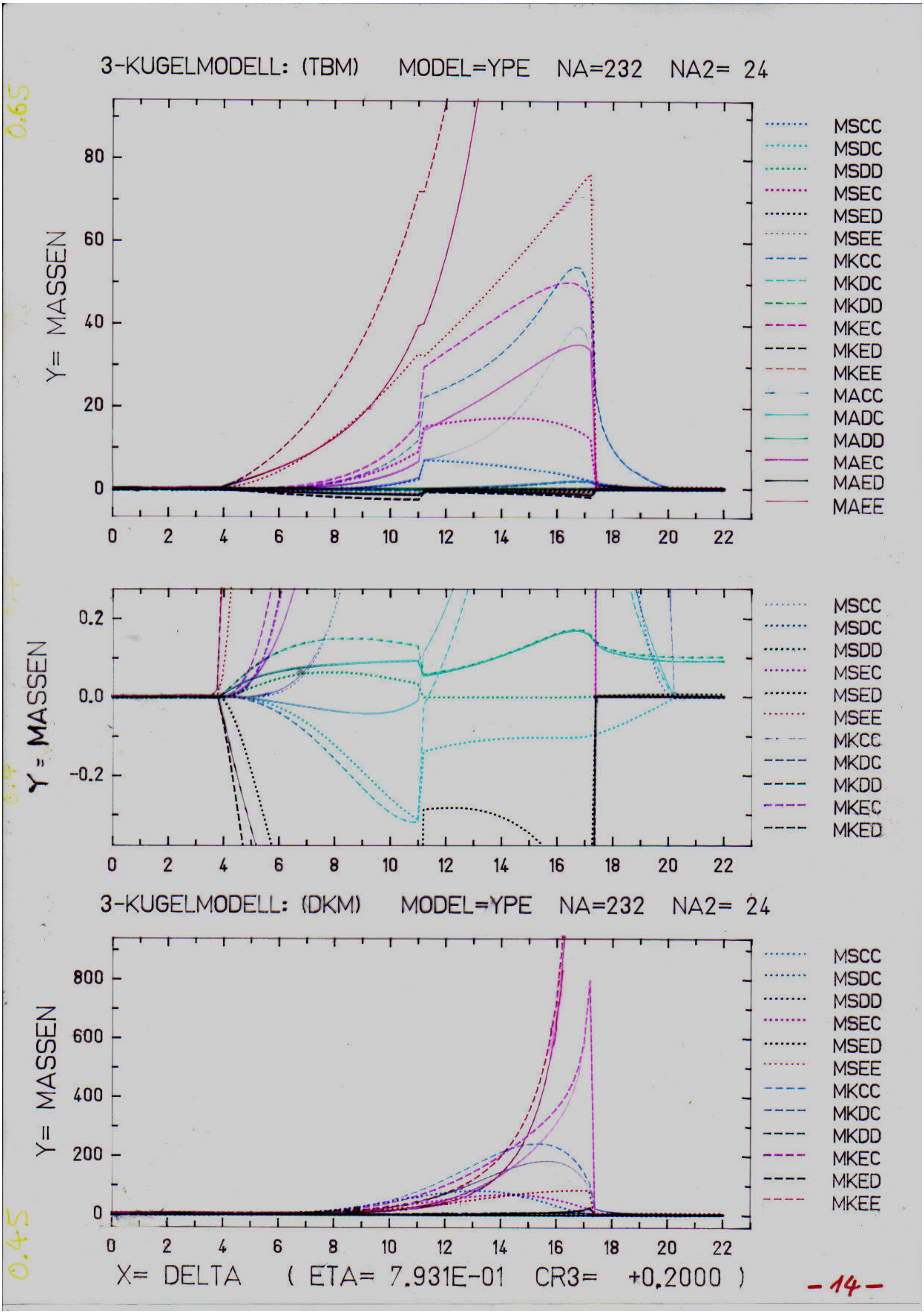
$$F_4 = (R2 + SR3 \cdot R3) \cdot SIN2 - \rho_3 = 0$$

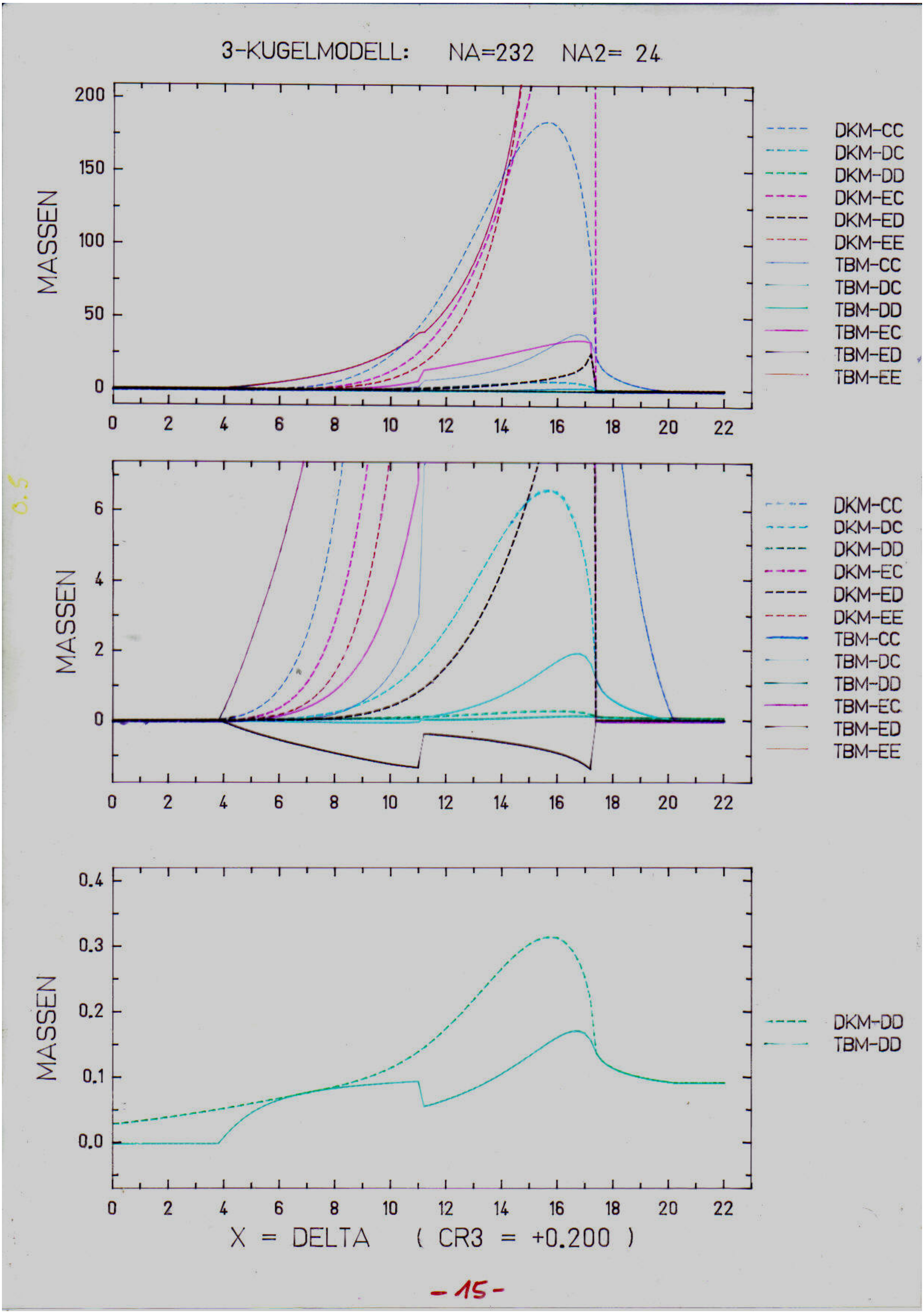
And the length condition gives:

$$F_5 = (R1+SR3 \cdot R3) \cdot KOS1 + (R2+SR3 \cdot R3) \cdot KOS2 - DELTA = 0$$

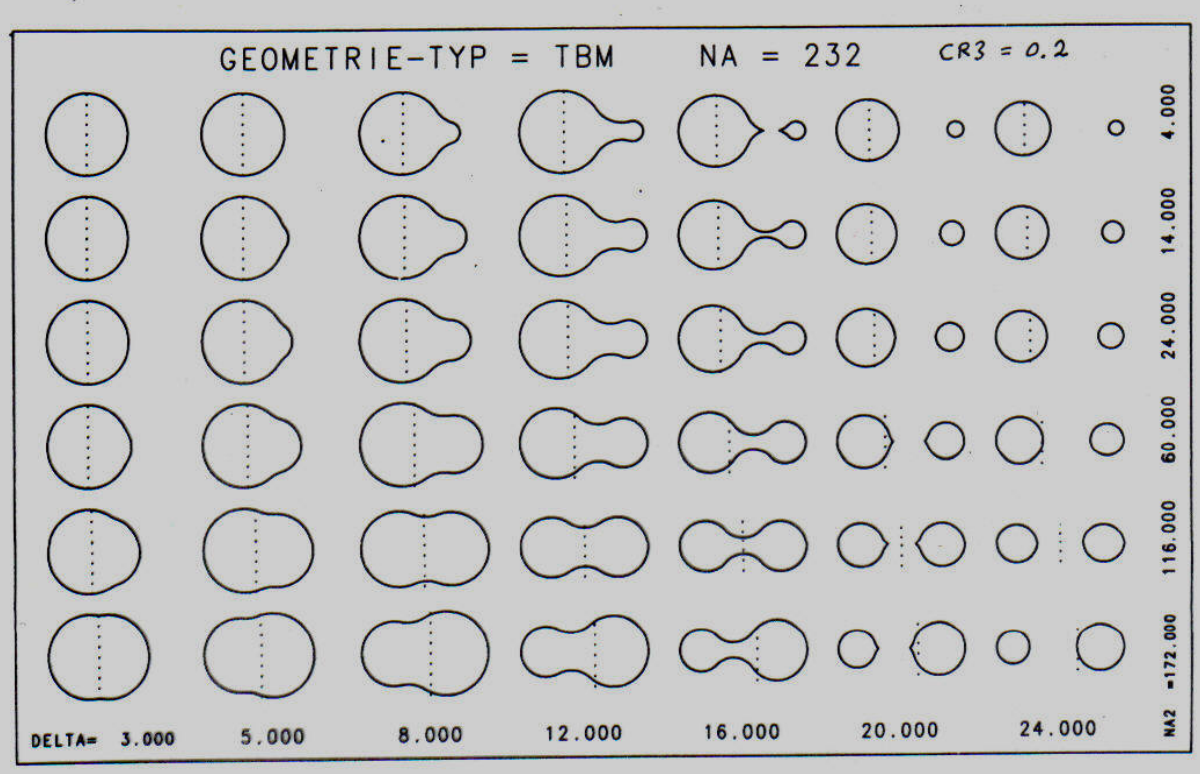
-12-


3-KUGELMODELL: (TBM) MODEL=YPE NA=232 NA2= 24
Y= GVONX
DELZB
DELZH
DELZV
DELZS
DELZF
V2/H
V1/H
KOS2
KOS1
R2F
R1F
Z3
RHO3
R3
C2
U2
U1
C1
R2
R1
3-KUGELMODELL: (DKM) MODEL=YPE NA=232 NA2= 24
Y= GVONX
DELZS
DELZF
V2/H
V1/H
KOS2
KOS1
R2F
R1F
Z3
RHO3
R3
C2
U2
U1
C1
R2
R1
X= DELTA (ETA= 7.931E-01 CR3= +0.2000)
-13-

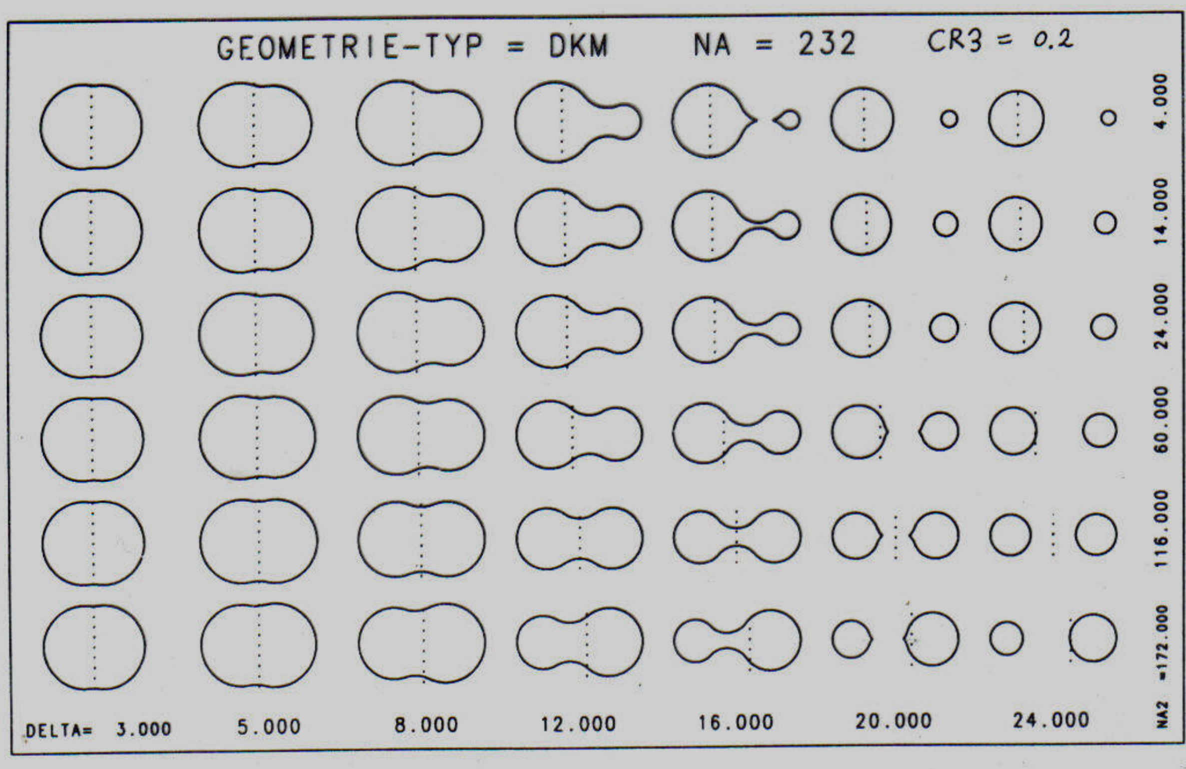

3-KUGELMODELL: (TBM) MODEL=YPE NA=232 NA2= 24
Y= MASSEN
MSCC
MSDC
MSDD
MSEC
MSED
MSEE
MKCC
MKDC
MKDD
MKEC
MKED
MKEE
MACC
MADC
MADD
MAEC
MAED
MAEE
Y = MASSEN
MSCC
MSDC
MSDD
MSEC
MSED
MSEE
MKCC
MKDC
MKDD
MKEC
MKED
3-KUGELMODELL: (DKM) MODEL=YPE NA=232 NA2= 24
Y= MASSEN
MSCC
MSDC
MSDD
MSEC
MSED
MSEE
MKCC
MKDC
MKDD
MKEC
MKED
MKEE
X= DELTA (ETA= 7.931E-01 CR3= +0.2000)
-14-


3-KUGELMODELL: NA=232 NA2= 24
MASSEN
X = DELTA (CR3 = +0.200)
DKM-CC
DKM-DC
DKM-DD
DKM-EC
DKM-ED
DKM-EE
TBM-CC
TBM-DC
TBM-DD
TBM-EC
TBM-ED
TBM-EE
0.5
-15-

TBM - shapes in the DELTA / NA2 - plane

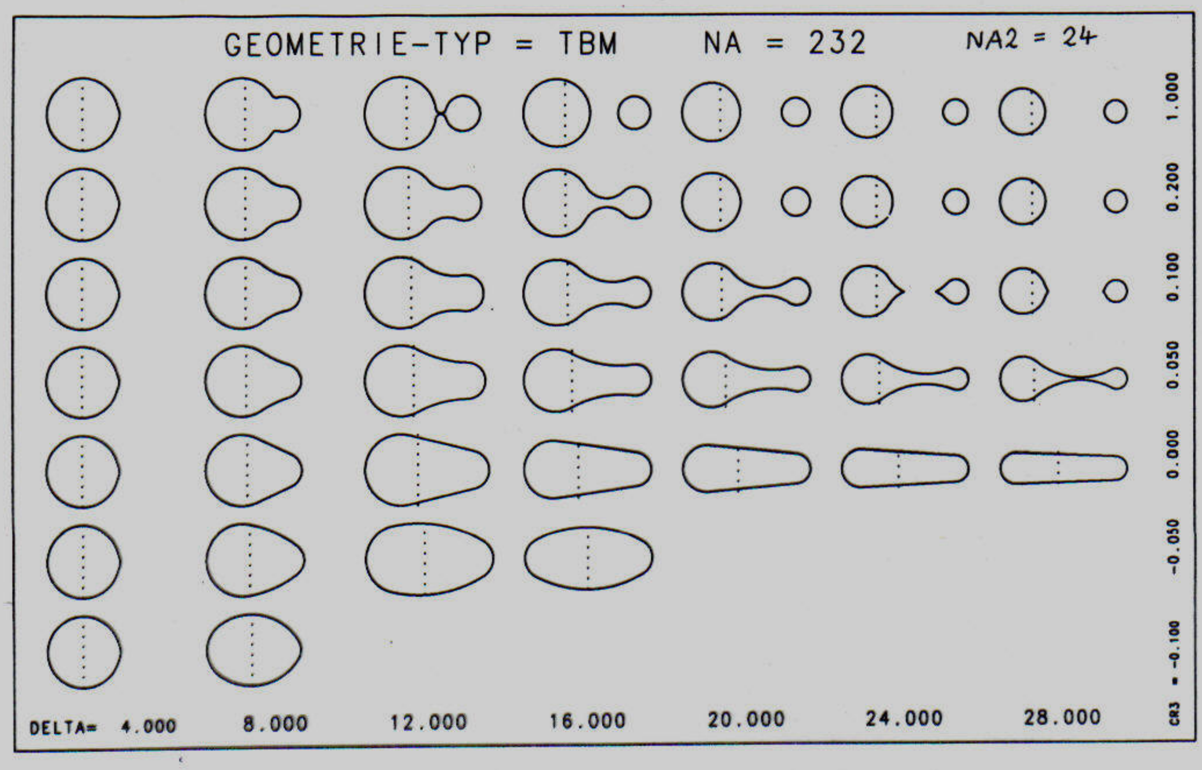


DKM - shapes in the DELTA / NA2 - plane

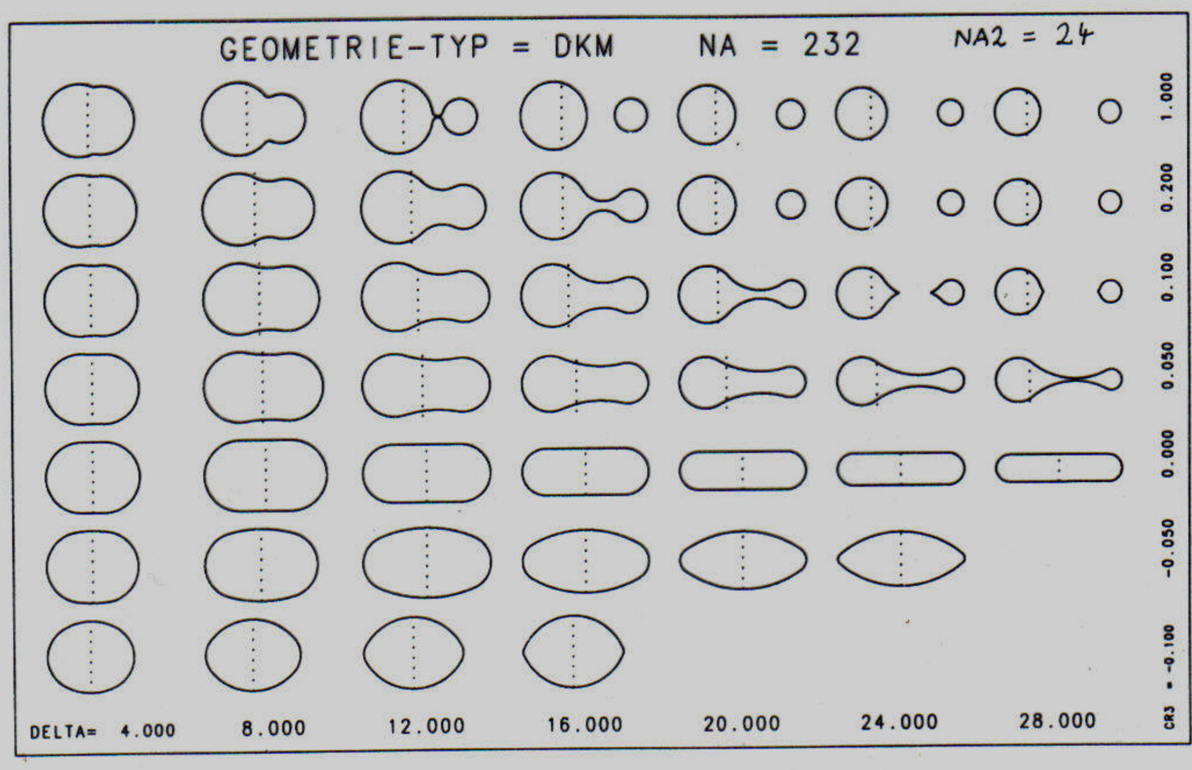


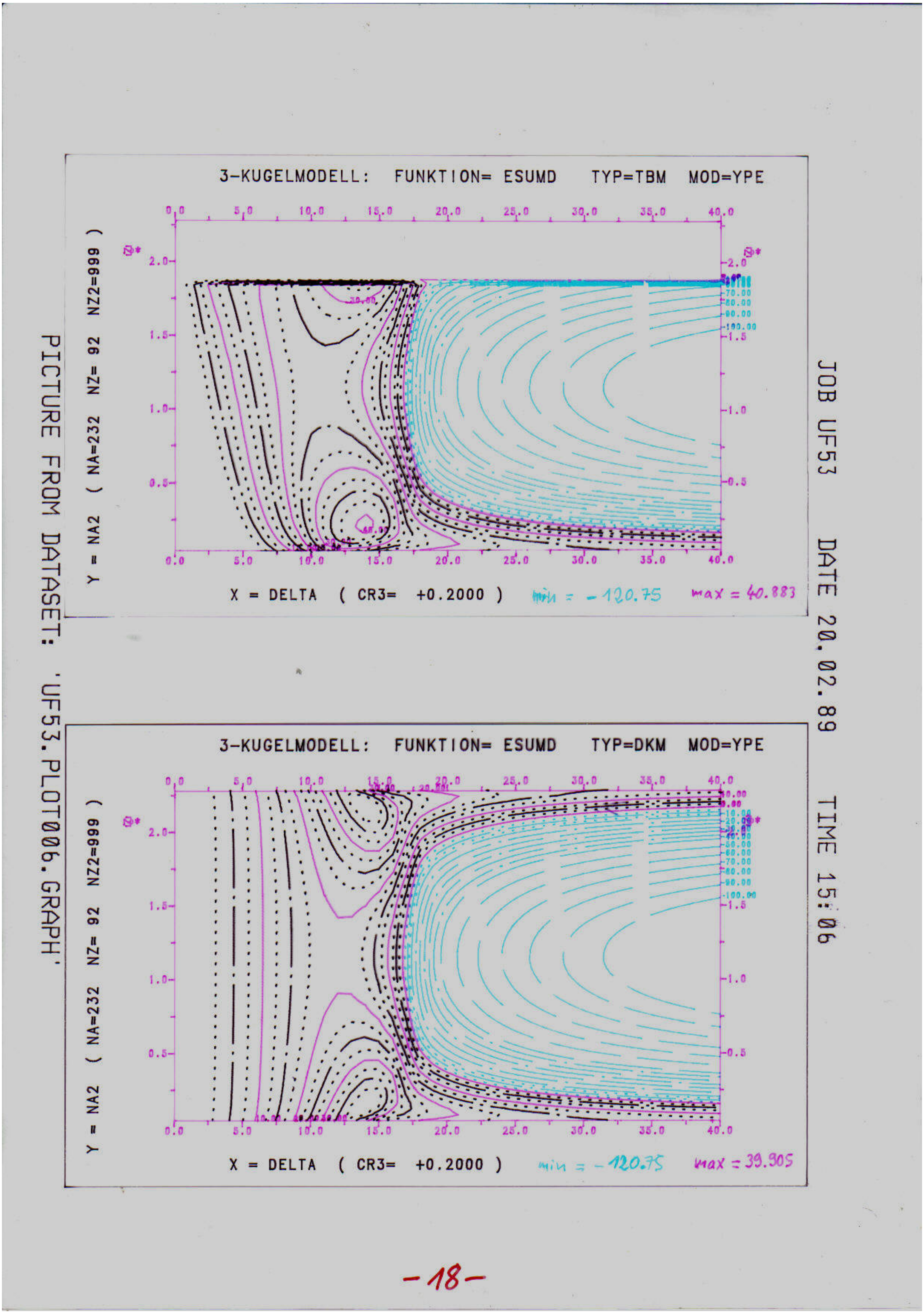
– 16 –

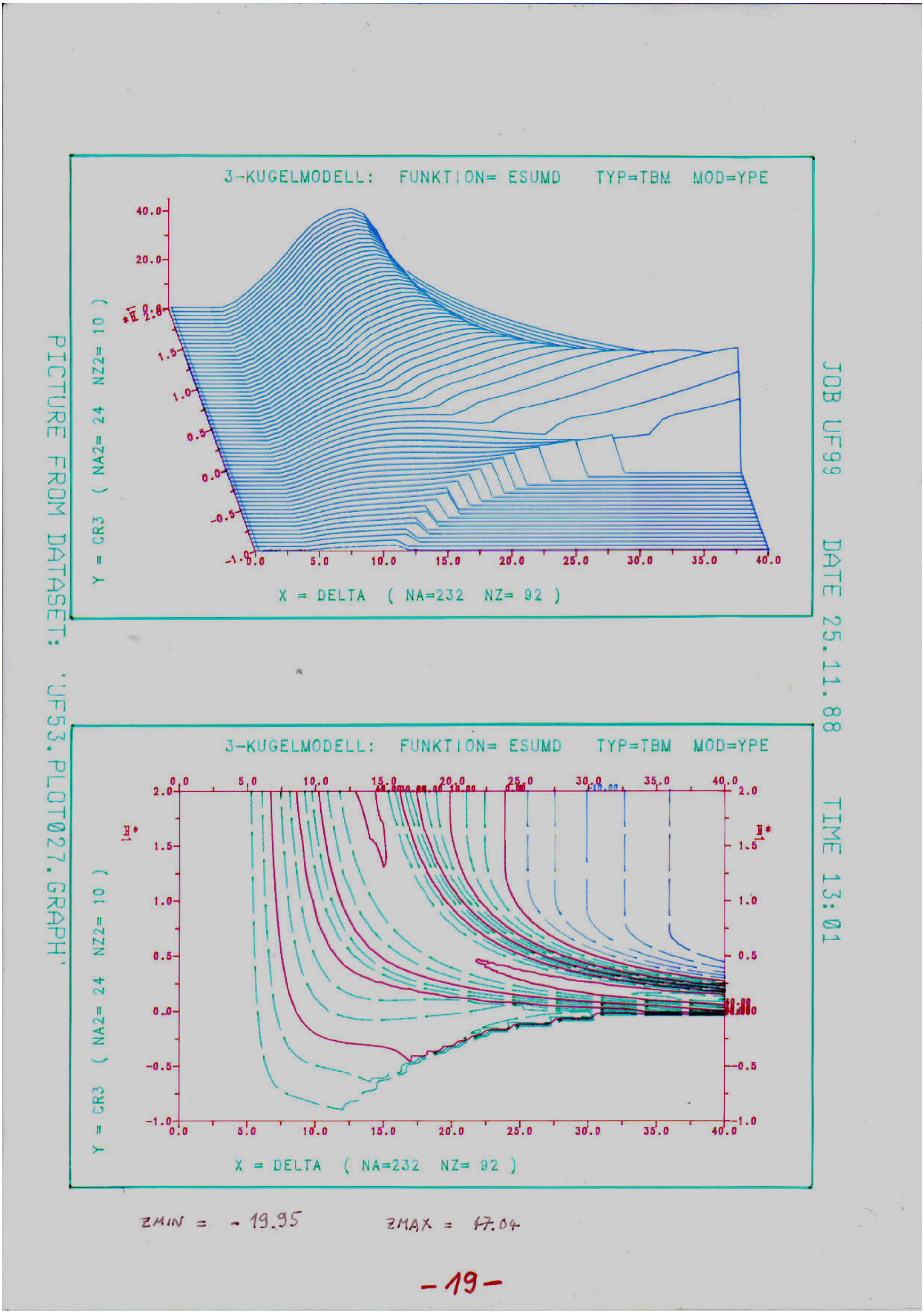
TBM - shapes in the DELTA / CR3 plane

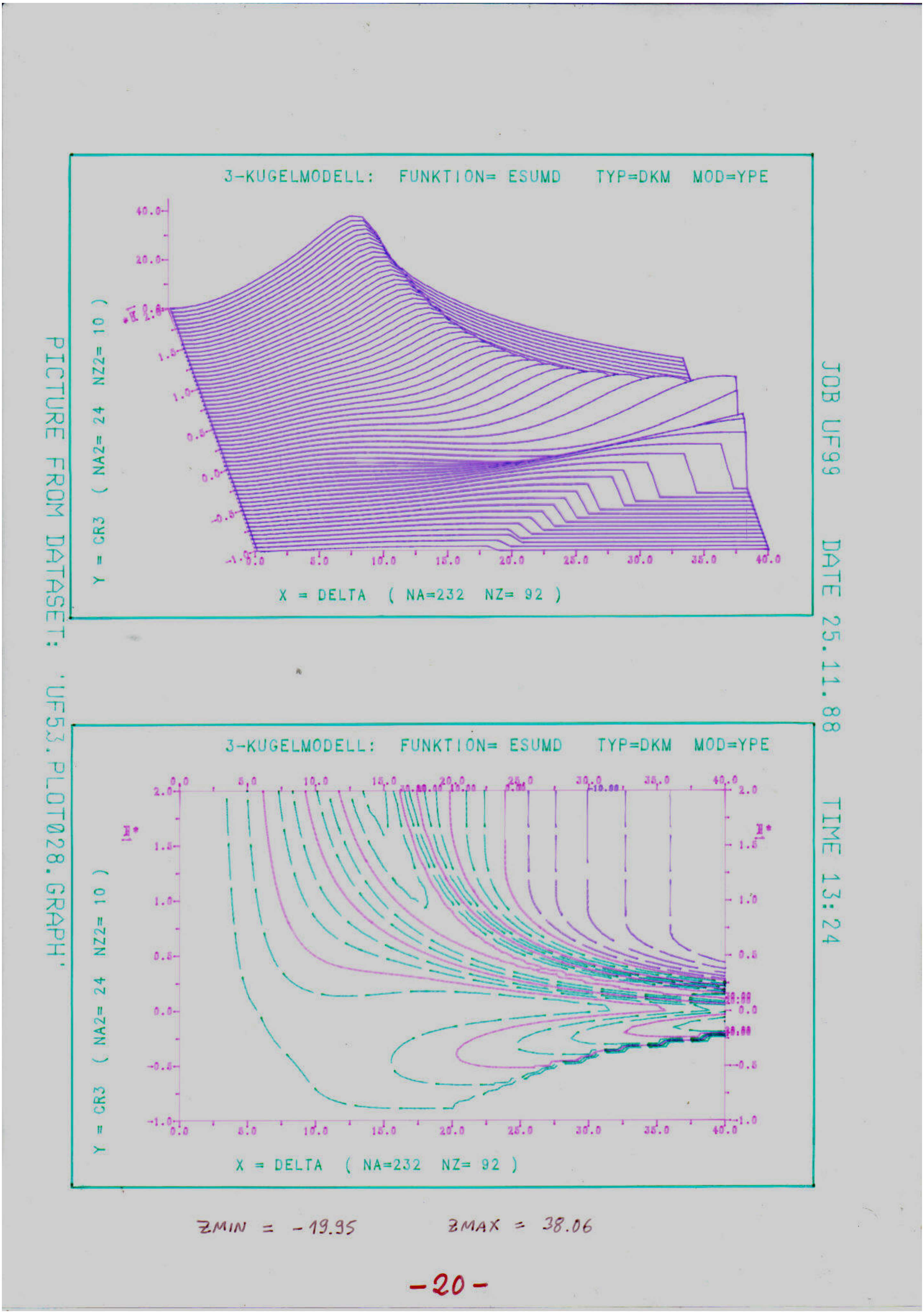


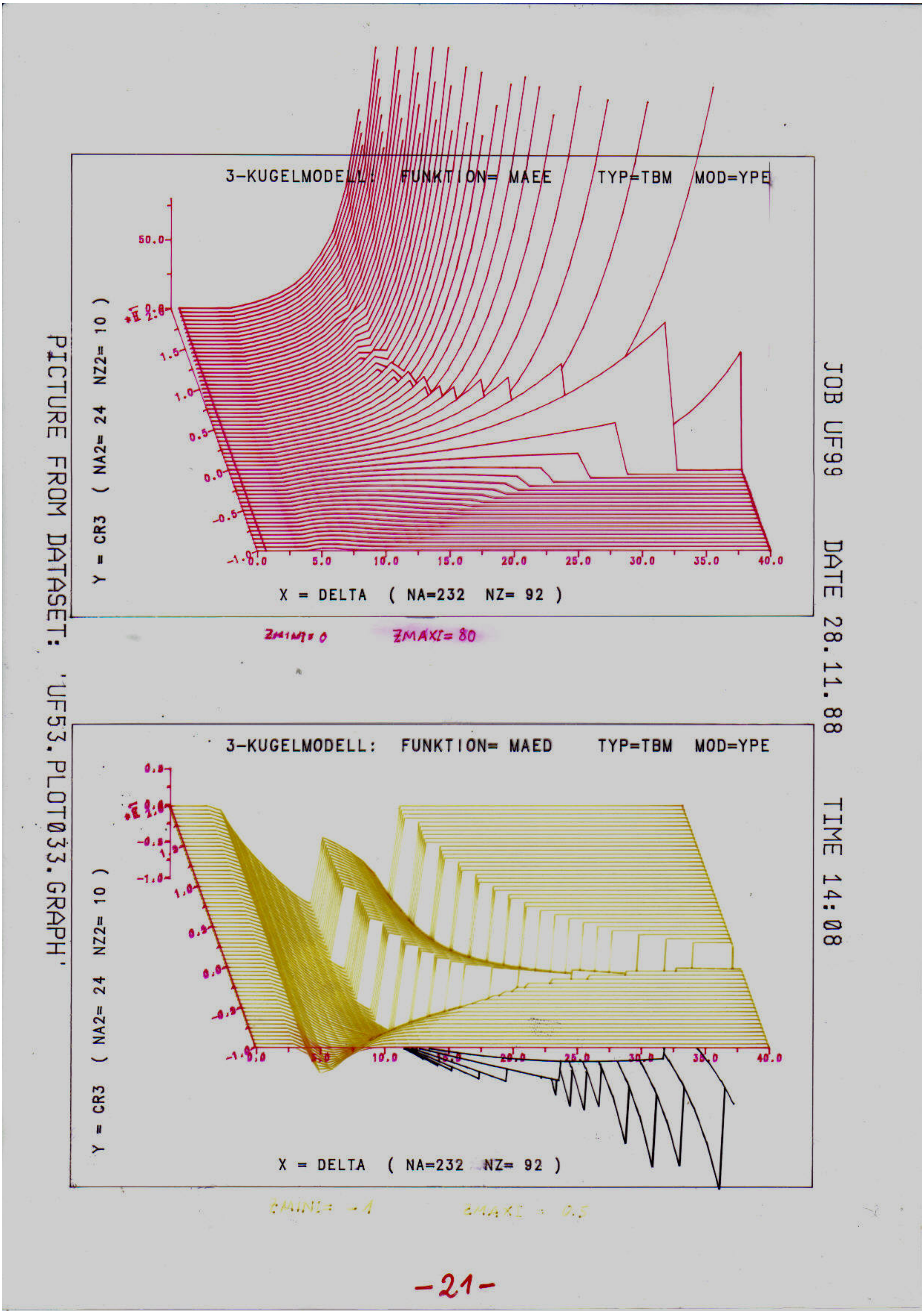
DKM - shapes in the DELTA / CR3 plane

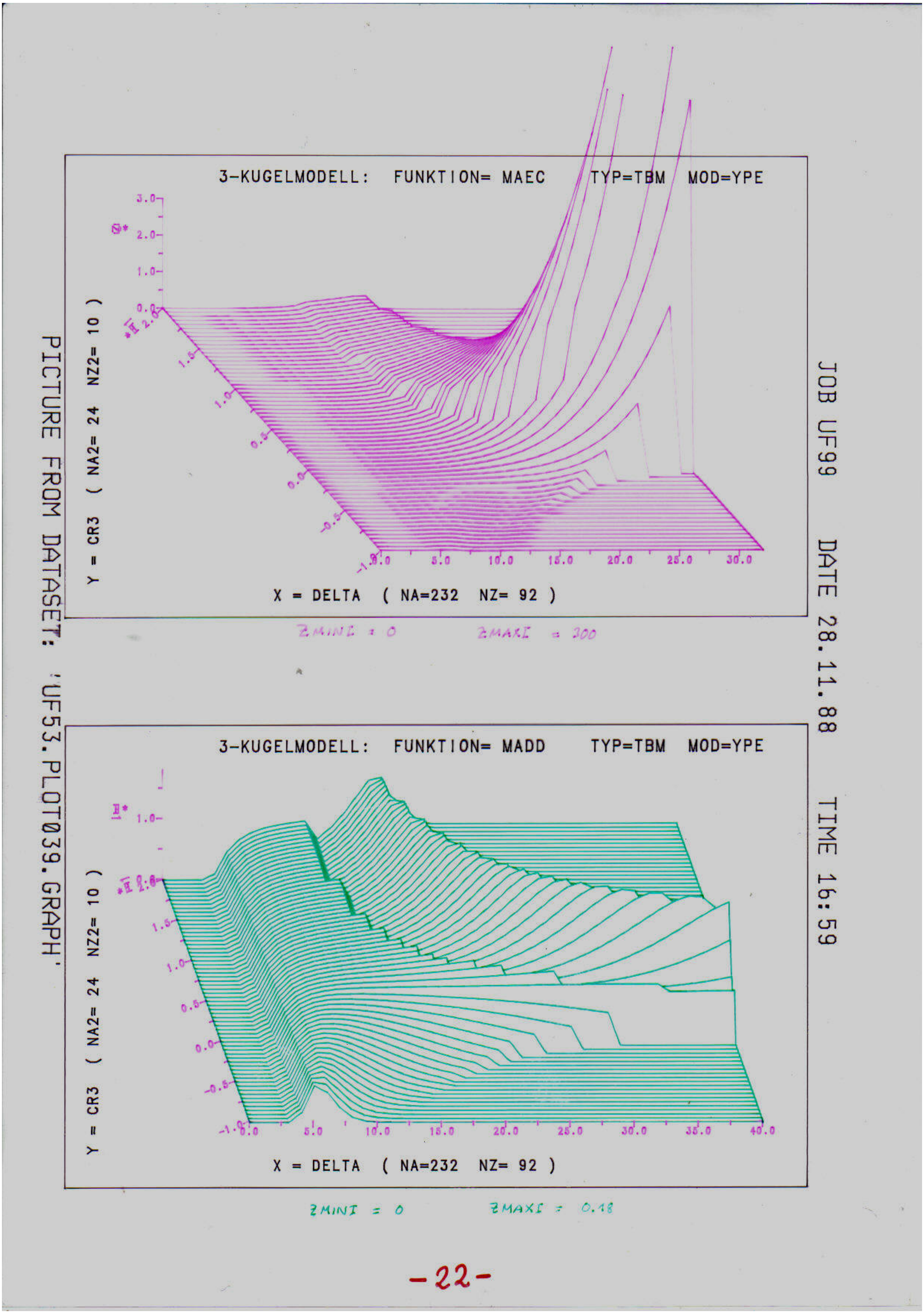


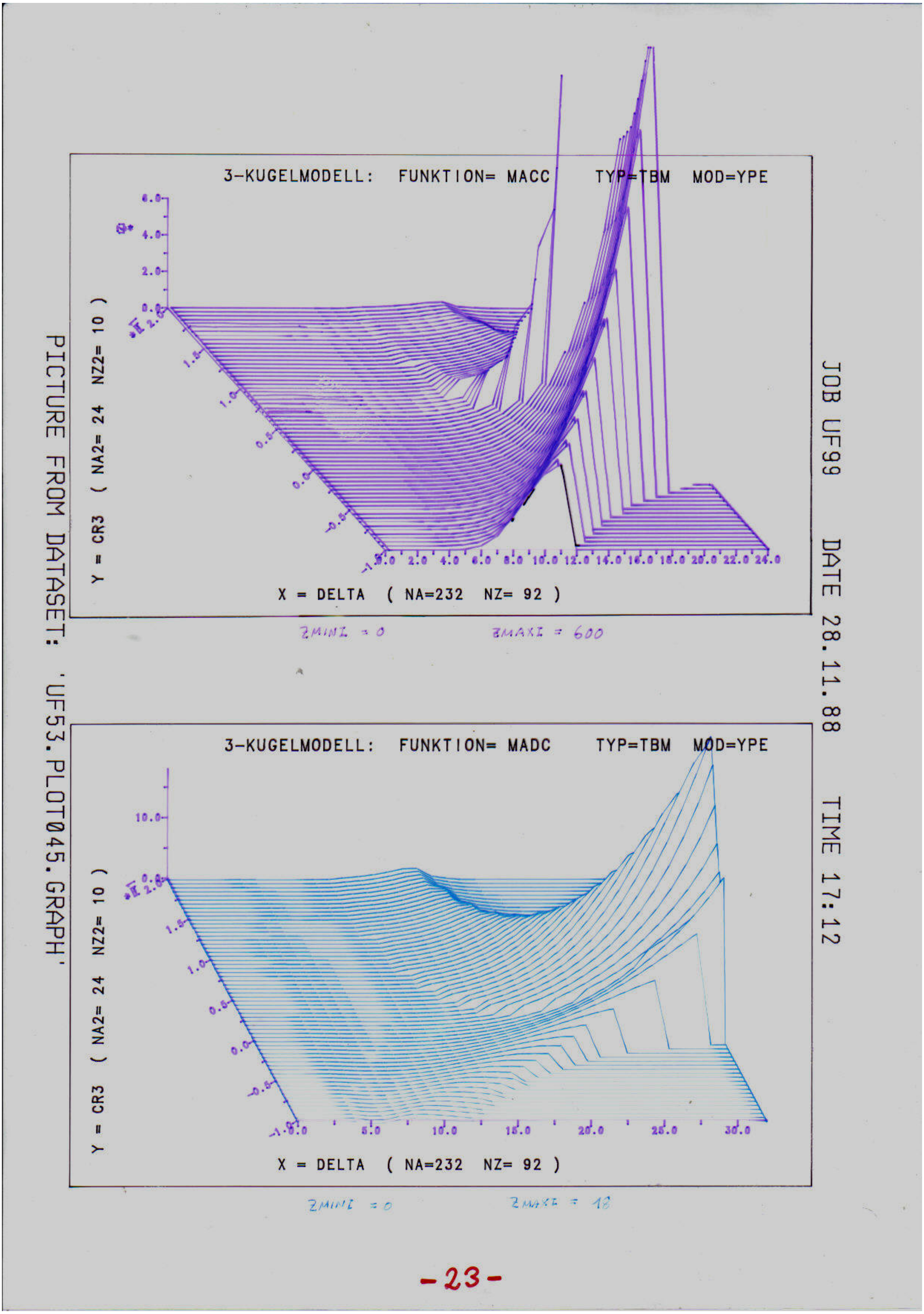

3-KUGELMODELL: FUNKTION= ESUMD TYP=TBM MOD=YPE
Y = NA2 (NA=232 NZ= 92 NZ2=999)
X = DELTA (CR3= +0.2000)
min = -120.75
max = 40.883
3-KUGELMODELL: FUNKTION= ESUMD TYP=DKM MOD=YPE
Y = NA2 (NA=232 NZ= 92 NZ2=999)
X = DELTA (CR3= +0.2000)
min = -120.75
max = 39.905
PICTURE FROM DATASET: 'UF53.PLOT006.GRAPH'
JOB UF53 DATE 20.02.89 TIME 15:06
-18-

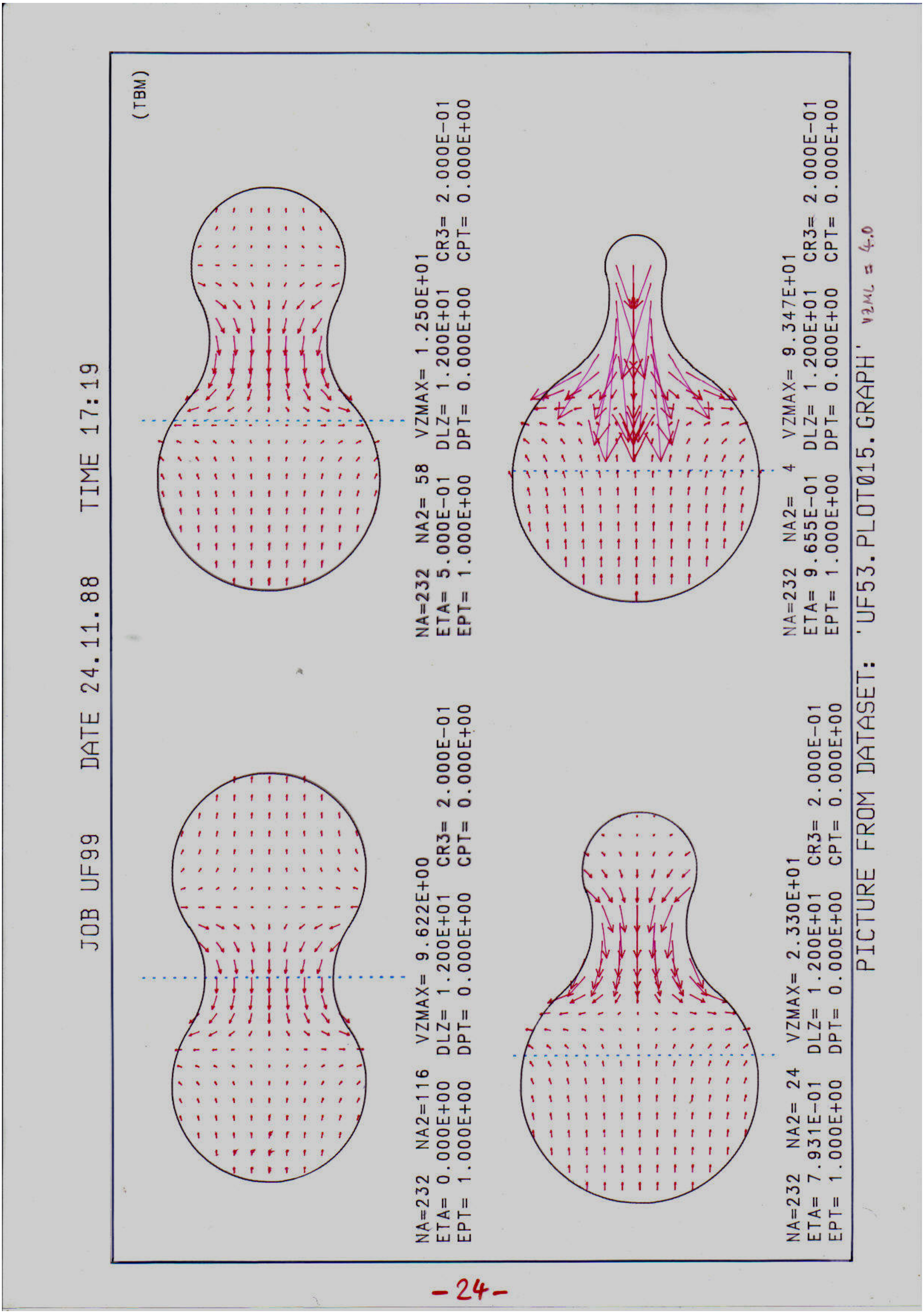

3-KUGELMODELL: FUNKTION= ESUMD TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
3-KUGELMODELL: FUNKTION= ESUMD TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
PICTURE FROM DATASET: 'UF53.PLOT027.GRAPH'
JOB UF99 DATE 25.11.88 TIME 13:01
ZMIN = - 19.95
ZMAX = 17.04
-19-

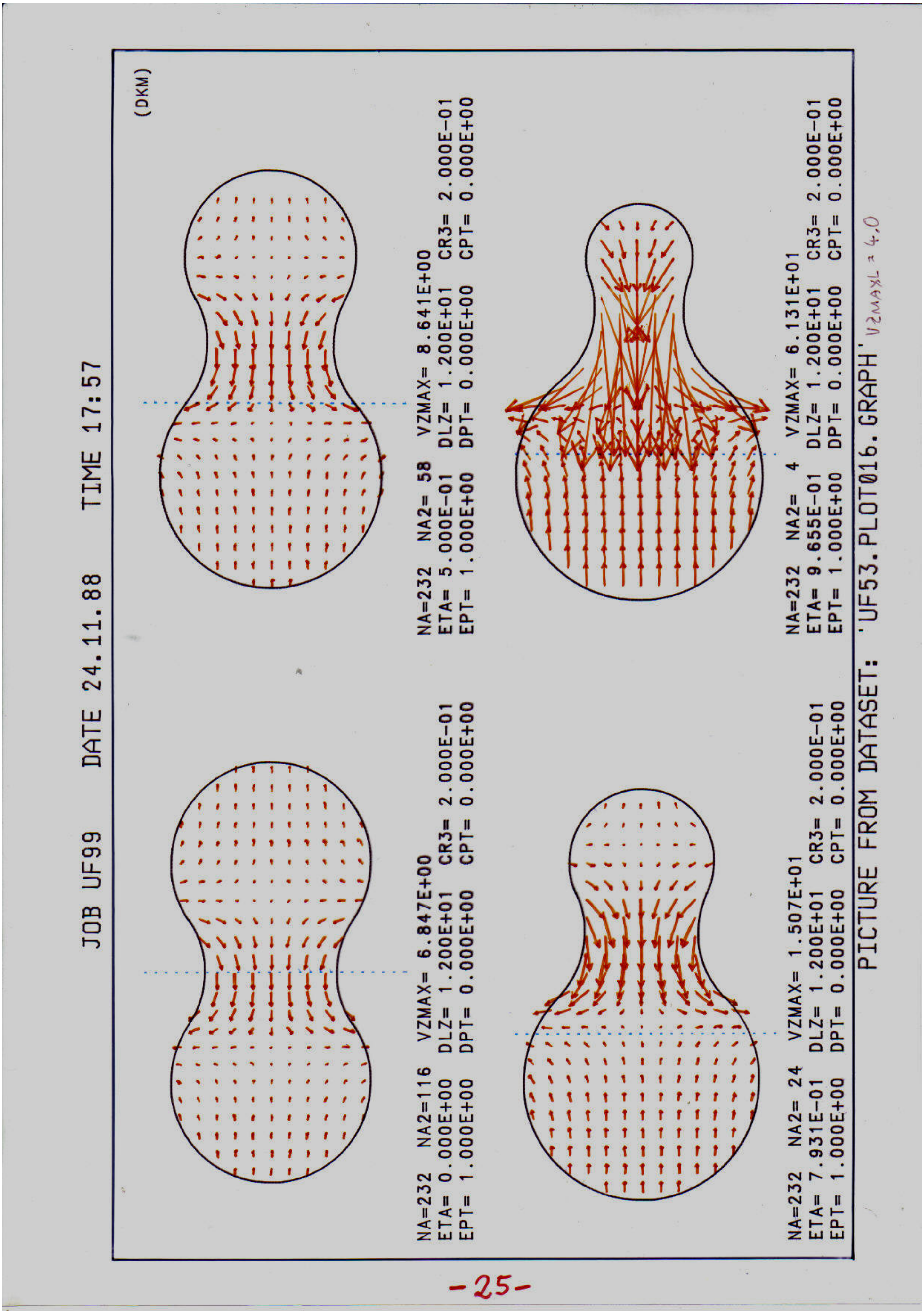

3-KUGELMODELL: FUNKTION= ESUMD TYP=DKM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
3-KUGELMODELL: FUNKTION= ESUMD TYP=DKM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
PICTURE FROM DATASET: 'UF53.PLOT028.GRAPH'
JOB UF99 DATE 25.11.88 TIME 13:24
ZMIN = -19.95
ZMAX = 38.06
-20-

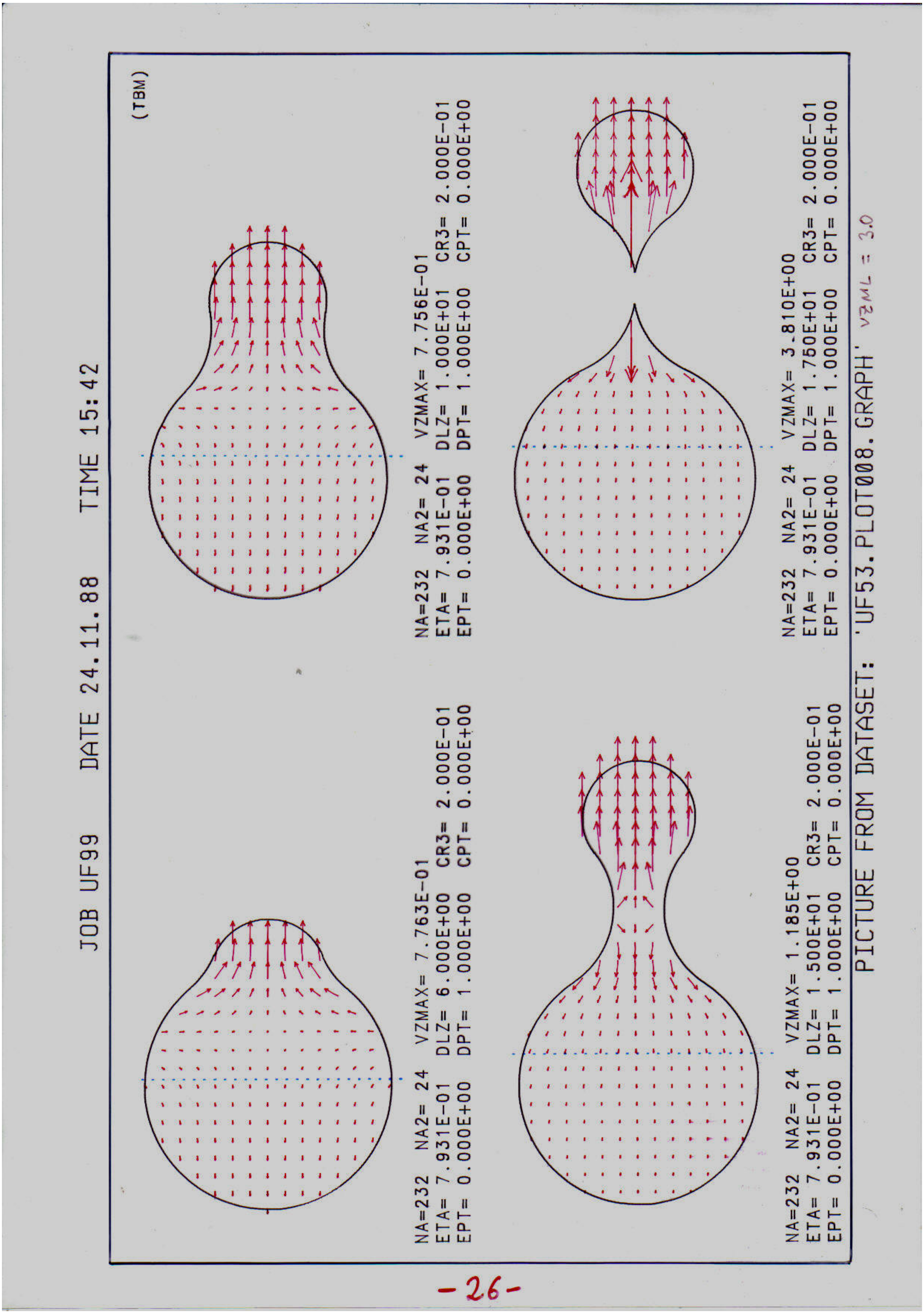

PICTURE FROM DATASET: 'UF53.PLOT033.GRAPH'
JOB UF99
DATE 28.11.88
TIME 14:08
3-KUGELMODELL: FUNKTION= MAEE TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
ZMINI= 0
ZMAXI= 80
3-KUGELMODELL: FUNKTION= MAED TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
-21-

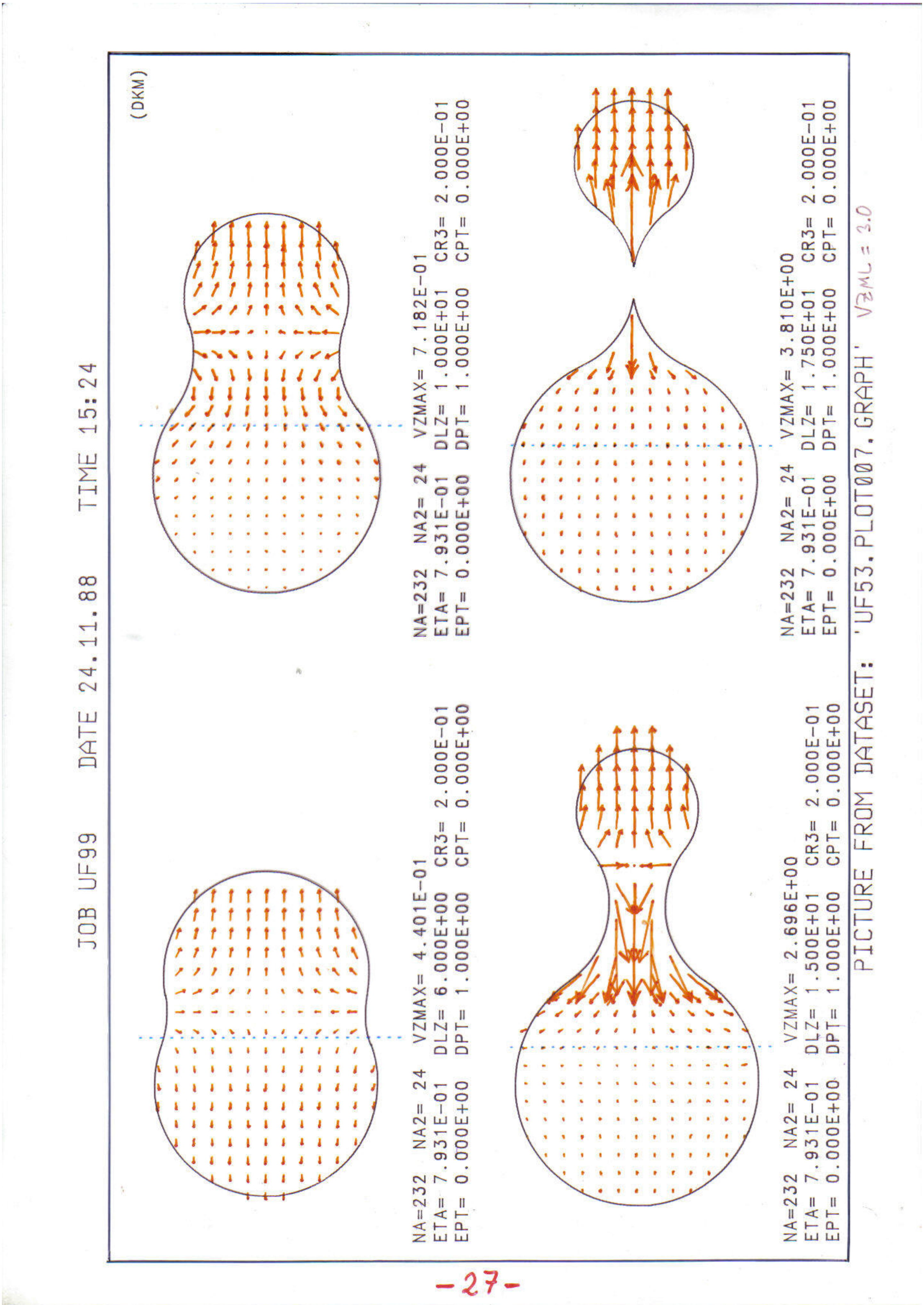

3-KUGELMODELL: FUNKTION= MAEC TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
ZMINI = 0
ZMAXI = 300
3-KUGELMODELL: FUNKTION= MADD TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
ZMINI = 0
ZMAXI = 0.18
PICTURE FROM DATASET: 'UF53.PLOT039.GRAPH'
JOB UF99 DATE 28.11.88 TIME 16:59
-22-

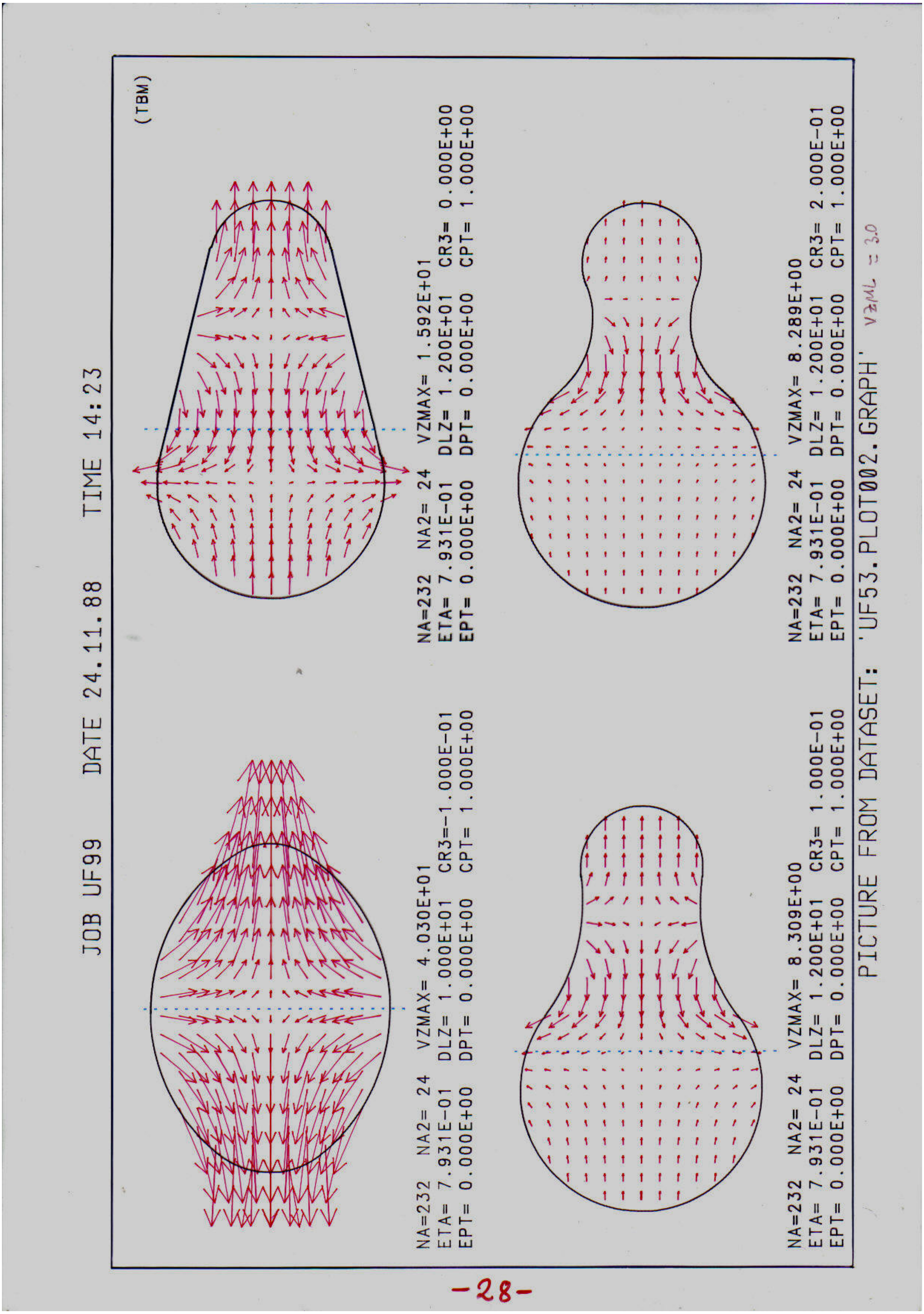

3-KUGELMODELL: FUNKTION= MACC TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
ZMINI = 0
ZMAXI = 600
3-KUGELMODELL: FUNKTION= MADC TYP=TBM MOD=YPE
Y = CR3 (NA2= 24 NZ2= 10)
X = DELTA (NA=232 NZ= 92)
ZMINI = 0
ZMAXI = 18
PICTURE FROM DATASET: 'UF53.PLOT045.GRAPH'
JOB UF99 DATE 28.11.88 TIME 17:12
-23-

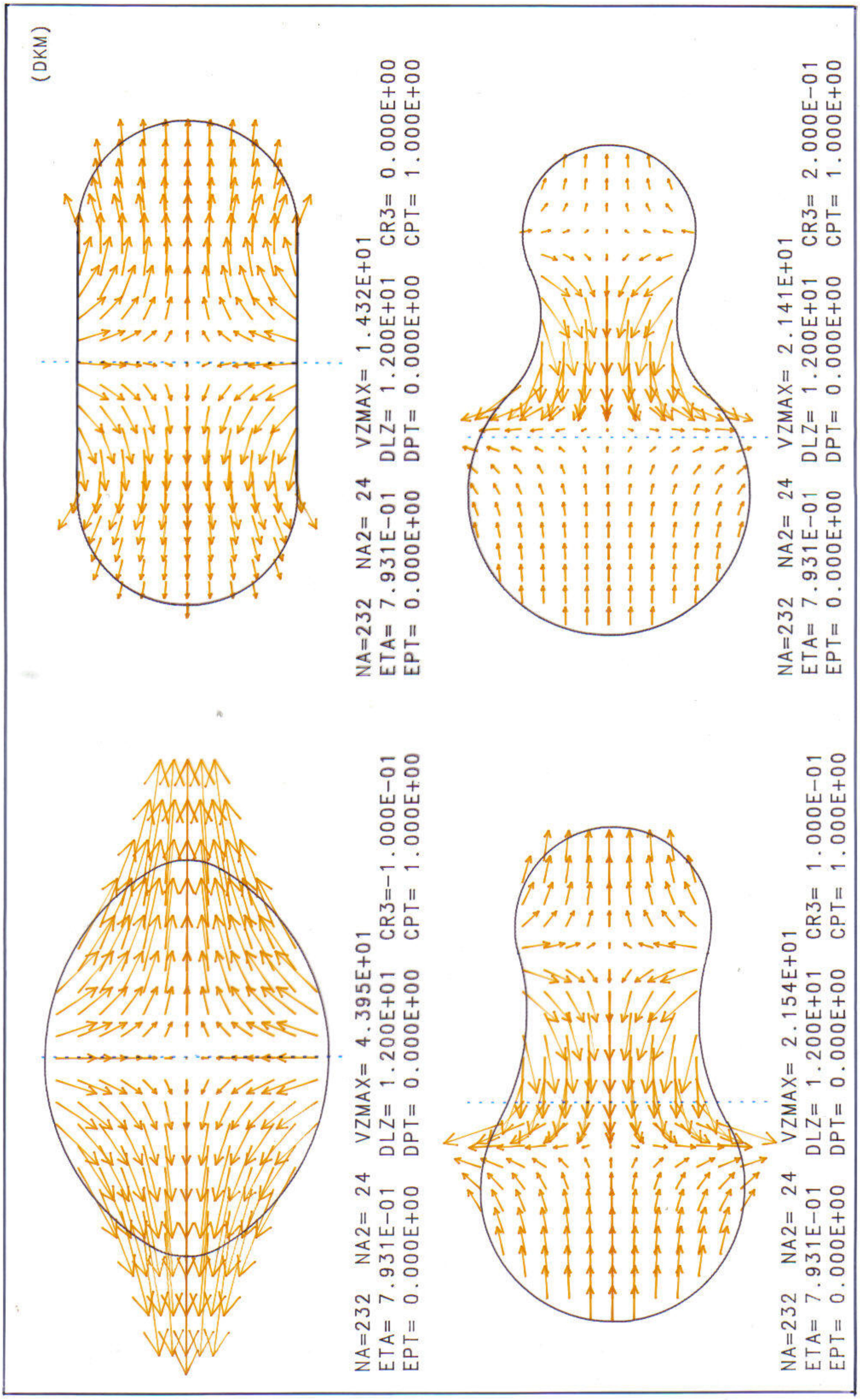

JOB UF99 DATE 24.11.88 TIME 17:19
(TBM)
NA=232 NA2=116 VZMAX= 9.622E+00
ETA= 0.000E+00 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 58 VZMAX= 1.250E+01
ETA= 5.000E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 2.330E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 4 VZMAX= 9.347E+01
ETA= 9.655E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
PICTURE FROM DATASET: 'UF53.PLOT015.GRAPH'
-24-


JOB UF99 DATE 24.11.88 TIME 17:57
(DKM)
NA=232 NA2=116 VZMAX= 6.847E+00
ETA= 0.000E+00 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 58 VZMAX= 8.641E+00
ETA= 5.000E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 1.507E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
NA=232 NA2= 4 VZMAX= 6.131E+01
ETA= 9.655E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 1.000E+00 DPT= 0.000E+00 CPT= 0.000E+00
PICTURE FROM DATASET: 'UF53.PLOT016.GRAPH'
VZMAXL = 4.0
-25-


JOB UF99 DATE 24.11.88 TIME 15:42
(TBM)
NA=232 NA2= 24 VZMAX= 7.763E-01
ETA= 7.931E-01 DLZ= 6.000E+00 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 7.756E-01
ETA= 7.931E-01 DLZ= 1.000E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 1.185E+00
ETA= 7.931E-01 DLZ= 1.500E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 3.810E+00
ETA= 7.931E-01 DLZ= 1.750E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
PICTURE FROM DATASET: 'UF53.PLOT008.GRAPH'
VZML = 3.0
-26-


JOB UF99 DATE 24.11.88 TIME 15:24
(DKM)
NA=232 NA2= 24 VZMAX= 4.401E-01
ETA= 7.931E-01 DLZ= 6.000E+00 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 7.182E-01
ETA= 7.931E-01 DLZ= 1.000E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 2.696E+00
ETA= 7.931E-01 DLZ= 1.500E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
NA=232 NA2= 24 VZMAX= 3.810E+00
ETA= 7.931E-01 DLZ= 1.750E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 1.000E+00 CPT= 0.000E+00
PICTURE FROM DATASET: 'UF53.PLOT007.GRAPH'
-27-


JOB UF99 DATE 24.11.88 TIME 14:23
(TBM)
NA=232 NA2= 24 VZMAX= 4.030E+01
ETA= 7.931E-01 DLZ= 1.000E+01 CR3=-1.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 1.592E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 0.000E+00
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 8.309E+00
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 1.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 8.289E+00
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
PICTURE FROM DATASET: 'UF53.PLOT002.GRAPH'
-28-


JOB UF99 DATE 24.11.88 TIME 14:42
(DKM)
NA=232 NA2= 24 VZMAX= 4.395E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3=-1.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 1.432E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 0.000E+00
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 2.154E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 1.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
NA=232 NA2= 24 VZMAX= 2.141E+01
ETA= 7.931E-01 DLZ= 1.200E+01 CR3= 2.000E-01
EPT= 0.000E+00 DPT= 0.000E+00 CPT= 1.000E+00
PICTURE FROM DATASET: 'UF53.PLOT001.GRAPH'
VZML = 3.0

-29-